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A Unified Theory of DESIGN STRUCTURE MATRIX AND AXIOMATIC DESIGN FOR PRODUCT ARCHITECTURE

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ABSTRACT

We propose a new principle of modularity for product design in cases where functions and constraints contribute to modularity. A modularity matrix is defined as a set of permissible ranges for both functions and constraints. We prove that a design structure matrix (DSM) of physical components for functions is equivalent to a modularity matrix of axiomatic design (AD). Next, from statistical mechanics, we introduce entropy for selected modules with constraints after regularizing a modularity matrix by a new parameter. This concept of entropy highlights the number of possible options to realize the optimal value of a product. We show that the two limits of the entropy are logically equivalent to modularizations of real option theory and information axiom in DSM and AD. From the principle of entropy maximization, we clarify the manner in which the matrix elements of modularity matrix should change in order to realize the highest value of products.

1. Introduction

Modularity has been explored as a useful method to classify designs, products, and industries by the use of technology. Modularity is defined by various types of modularity matrices on base vector spaces such as designs, tasks, organizations, functions, components, products, and firms, for example *(Krishnan and Ulrich, 2001; Browning, 2001; Eppinger and Browning 2012)*. A modularity matrix has been mainly defined in two ways. The first definition includes the concept of a design structure matrix *(DSM)*, which was invented by Steward *(1981)*, developed by Eppinger *(1991)*, and formulated by Baldwin and Clark *(2000)* both to analyze and manage the complex systems. The DSM is a mapping between the same base vector space to visualize the structures of design systems *(an example of which is found in* **Figure 1** *when modules 2 and 4 are exchanged)*. Interactions within a DSM signify the transfer of material, energy, and information, which are defined in abstract terms and remain beyond the reach of facile mathematical definitions. We find many applications of DSM, for examples product, organization and process architectures in the reviews *(Browning, 2001, Eppinger and Browning, 2012)*. Recent studies have analyzed the relationships among DSMs across different base vector spaces, which are referred to as domain mapping matrices *(DMMs)* and multiple domain matrices *(MDM) (Jacobides and Winter 2005; Danilovic and Browning 2007; Baldwin, 2008, Lindemann et al., 2009; Luo et al., 2009)*.

$$2 \Leftrightarrow 4$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \qquad \begin{array}{cccc} 1 & 4 & 3 & 2 \end{array}$$

$$\begin{pmatrix} X & & & \\ & X & X & X \\ X & & X & \\ X & & & X \end{pmatrix} \quad \begin{pmatrix} X & & & \\ X & X & & \\ X & & X & \\ & X & X & X \end{pmatrix}$$

FIGURE 1: *Example of a DSM when exchanging the order of modules*

The second definition was formulated by Ulrich *(1995)* and includes a shift in emphasis from mapping physical components to functional requirements; this approach was theoretically systematized by Suh *(2001)* as a general framework to analyze the interplay between the different vector spaces, and can be called axiomatic design theory *(AD)*. Interaction in AD is defined by a mathematical approach to the mapping of the physical components and functional requirements in order to optimize product design. We find many applications of AD, for examples product, software, supply chain, manufacturing systems in the reviews *(Krishnan et.al., 2001, Kulak et.al., 2010)*. Although these two theories *(DSM and AD)* have been developed independently towards almost the same goal, a crucial problem is that the relationship of DSM and AD, as well as the differences of their models, largely remains unclear, despite that Dong and Whitney *(2001)* and Tang et al. *(2009)* have addressed the topic. One objective of this paper is to clarify equivalence of the modularity matrices of DSM and AD and the applicable ranges of these theories.

Another problem tackled in this paper is how constraints can change modularity. Previous studies on the modularity theory often neglect the topic of constraints for products. In DSM, constraints must implicitly be imposed to realize products; in AD, constraints cannot be systematically included within the theory. Three types of constraints—physical, equipment, and operational constraints—are essential for the technological realization of products *(Fujimura, 2000)*. In this paper, we construct a modularity matrix for constraints as well as functions, which is useful for the study of how constraints contribute to modularity.

Moreover, we introduce a way to regularize a modularity matrix by means of a new parameter, entailing a capability to find unknown and permissible ranges for functions and constraints. Entropy for the regularized modularity matrix can be introduced with statistical mechanics. Here the concept of entropy points to the number of possible options within a system in order to realize the optimal value of products. We propose a new principle of modularity based on the principle of entropy maximization, which increases the value or performance of products. We verify our theory by the fact that two different limits of this entropy are logically equivalent to the modularizations *(real option theory and information axiom)* of DSM and AD. Finally, we clarify how the principle of modularity can be used to understand changes in order to realize the optimal performance of products.

The organization of this paper is as follows. In Section 2, we briefly review DSM, AD, and three types of constraints. In Section 3, we propose a new definition of a modularity matrix for functions and constraints and prove equivalence between the modularity matrices of DSM and AD. In Section 4, we introduce a new parameter in order to regularize the modularity matrix and utilize statistical mechanics to propose new principles of modularity based on the principle of entropy maximization. Section 5 includes conclusions and future works.

2. Literature review

In this section, we briefly review DSM, AD, and three types of constraints.

The first item for review is DSM. Historically, Steward *(1981)* developed the fundamental theory of DSM and Eppinger *(1991)* applied DSM to concrete cases of product design. By use of DSM, which provides the mappings of the same vector space as the design parameters *(DP)* in **Figure 2**, we can visualize the structures of systems for product design. In DSM, functions as well as constraints are implicitly satisfied to realize products. Matrix elements of DSM express the transfers of material, energy, and information among base vector space coordinates. As a definition of modularity, if the matrix elements of DSM are diagonal or block diagonal, then related products can be understood as "modular." If the upper triangular matrix elements of DSM are zero, then the products are "hierarchical." If the upper triangular matrix elements of DSM are not zero, then the products are "integral." Baldwin et al. *(2013)* defined modularity strictly by range, which is expressed by the use of a transitive closure of DSM to successive power. In this paper, we adopt the definition of modularity by Baldwin et al. *(2013)*. As applications to various products, several types of DSM have been proposed, including those addressing tasks, design parameters, physical components, firms, and sectors *(Browning, 2001, Eppinger and Browning, 2012)*. In this paper, DPs, physical components *(PCs)* and firms-sectors *(FSs)*, which represent a business ecosystem *(Baldwin, 2008)*, will be considered as base vector spaces of modularity matrices.

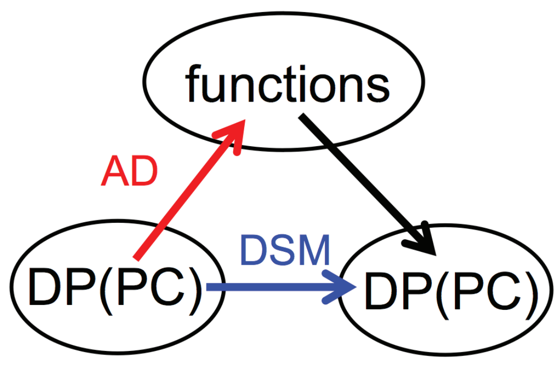

FIGURE 2: *Mapping DSM and AD*

Baldwin and Clark *(2000)* established the basic theory for DSM to introduce six complete modular operators such as splitting, substituting, augmenting, excluding, inverting, and porting. For modularization, Baldwin and Clark *(2000)* proposed that one factor to determine the modularity of products is to increase the number of options for designs by applying real option theory. Here, we briefly review the modularization of DSM. A distribution function *f(x)* of performance for a base vector space coordinate x is

$$f(x) = \frac{e^{-\frac{x^2}{\sigma^2}}}{\sqrt{2\pi}\sigma},$$

where σ is the standard deviation. A value of a product with one module V1 is

$$V_1 = \int_0^{\infty} dx \cdot x f(x) = \frac{\sigma}{\sqrt{2\pi}}.$$

When the standard deviation becomes larger, the value of the product increases linearly. For more general cases, we can consider Baldwin and Clark *(2000)*.

The second item for review is AD. Notably, AD evolved from the optimization theory in order to determine design parameters satisfying maximization, minimization, and permissible ranges for functions. AD is useful to classify relationships between functions, or functional requirements *(FRs)*, and PCs for product design. Here, the mapping of Ulrich *(1995)* is of interest *(in* **Figure 2***)*. Further details and the applications to various products are provided by Krishnan et.al.*(2001)* and Kulak et.al.*(2010)*. In AD, constraints to realize products have not been systematically introduced into the theory. Ulrich *(1995)* and Suh *(2001)* defined modularity from matrix elements of the mapping as FR and PC. When the matrix elements are diagonal, the products uncouple. When the upper triangular matrix elements are zero *(or not zero)*, the products decouple *(or coupled)*.

Suh *(2001)* proposed two axioms in AD for product design. The first is the independence axiom to maintain the independency of FR. Generally, this independence axiom is considered to be equivalent to maintain the modularity matrix of the mapping of FR PC to be uncoupled. The second is the information axiom to minimize the information content of product design, to realize the optimal design. Regarding the information axiom, the information content I is defined as a logarithm of probabilities $\tilde{P}_i$ for i=1,2,…,n, to realize the system such that

$$I = -\sum_{i=1}^{n} \log_2 \tilde{P}_i,$$

where n is the number of modules in one system. For further details, we can consider the work of Suh *(2001)*.

Third, we review three types of constraints such as physical constraints, equipment constraints, and operational constraints *(Fujimura, 2000)*. Constraints signify the conditions that must be satisfied in order to realize products. In **Figure 3**, we introduce a performance correlation diagram *(Fujimura, 2000)*, which shows a hierarchy of technology, wherein physical, equipment, operational constraints exhibit an ordered, nested relationship.

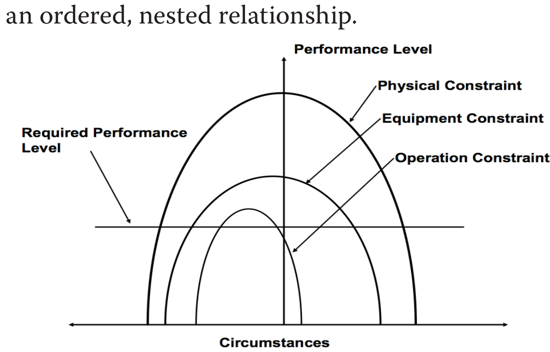

FIGURE 3: *Performance correlation diagram*

Physical constraints are conditions based on the fundamental law of science, and this includes physical and chemical phenomena. Equipment constraints are conditions dependent on the precision of the technological equipment made to realize products; this includes machines and materials. Operational constraints are conditions determined by establishing the parameters of working environments, such as ambient temperature and atmospheric pressure. This also includes a consideration of related circumstances ideal in order to realize products, including the use of time.

3. New Definition of Modularity Matrix

In this section, we propose a new definition of modularity matrix for functions and constraints and demonstrate the equivalence between the modularity matrices of AD and DSM for the functions on PCs, disregarding tasks and others.

In the past, many conceptual studies for product design were developed within the context of creative thinking, system requirement engineering, product design and so on. An example of creative thinking is the "Geneplore Model" — proposed by Finke et al. *(1992)* – which is a cycling process between the generation phase and the exploration phase for new product ideas under product constraints. An example of system requirement engineering is "requirement specification" — Loucopoulos and Karakostas *(1995)* — which constitutes an interrelated set of three requirements such as enterprise requirements, functional requirements, and non-functional requirements, some of which may be related to the product constraints. Thus, it would be essential to consider functional requirement as well as constraints in both of creative and incremental processes of new product development.

First, we define the modularity matrix for functions, whose matrix elements are parameterized by the permissible ranges of functions in order to realize a product. We consider only small transformations around the initial values of functions. Namely, for design parameters $\{X_i\}$ and functions $\{f_i(X)\}$, which are functions of design parameters, we treat only small transformations $\{dX_i\}$ and $\{df_i(X)\}$ *(Yassine and Falkenburg, 1999)*. We assume that the numbers of various base vector spaces, such as DP and FR, are the same as n in one system, whose modules are labeled by i=1,2,…n. Generally, the optimization theory for product design states to find a set of solutions to satisfy the following equations *(Michelena-Papalambros 1995, Fujimura 2000)*.

$$df_i(X) = \alpha_i,$$

where α_i are parameters for permissible ranges of {dfi}. Here, we emphasize that to develop product design can be interpreted as finding a set of the solutions.

We employ a useful idea to construct a modularity matrix from a Design Matrix, which is defined in terms of a mapping: DP→FR *(Dong and Whitney, 2001)*. A new definition of modularity matrix Gf_{ij} is proposed as a mapping {dXi}→{dXi} as follows.

$$Gf_{ij} = \sum_{k,m=1}^{n} \left(\frac{\partial f_k}{\partial X_i}\right)^{-1} \cdot Kf_{km} \cdot \left(\frac{\partial f_m}{\partial X_j}\right),$$

where $(\partial f_k / \partial X_i)^{-1}$ is the inverse matrix of $(\partial f_k / \partial X_i)$, and Kfkm is a diagonal matrix, whose matrix elements are parameters α_i for permissible ranges of $\{df_i\}$ such that $Kf_{km} = diag(\alpha_1, \alpha_2, \ldots, \alpha_n)$ for n modules. When the initial values of DP, denoted by $\{X0_i\}$, are chosen, the initial values of functions, denoted by $\{f_i(X0)\}$, and the sets of permissible ranges of {dfi} are determined uniquely. We note that the matrix $(\partial f_m / \partial X_j)$ and the inverse matrix depend only on the initial value $\{X0_i\}$. Moreover, each matrix element of Gf_{ij} can be considered as a set for the permissible ranges, and we express $\{Gf_{ij}\}$ as a set of Gf_{ij}.

Next, we define the modularity matrix Gc_{ij} for constraints as a mapping:$\{dX_i\}\rightarrow\{dX_i\}$ in a similar way to a modularity matrix for functions as follows.

$$Gc_{ij} = \sum_{k,m=1}^{n} \left(\frac{\partial C_k}{\partial X_i}\right)^{-1} \cdot Kc_{km} \cdot \left(\frac{\partial C_m}{\partial X_j}\right),$$

where Kc_{ij} is a diagonal matrix, whose matrix elements are parameters γ_i for permissible ranges of $\{dC_i\}$ such that $Kc_{ij} = diag(\gamma_1, \gamma_2, \ldots, \gamma_n)$.

Modularity matrix G_{ij} for both functions and constraints is defined as

$$\{G_{ij}\} = \sum_{k=1}^{n} \{Gf_{ik}\} \cap \{Gc_{kj}\},$$

where $\{G_{ij}\}$, $\{Gf_{ik}\}$ and $\{Gc_{kj}\}$ express sets for the permissible ranges. All matrix elements of $\{G_{ij}\}$ are defined by the intersections of two sets of matrix elements, $\{Gf_{ik}\}$ and $\{Gc_{kj}\}$. One main reason for this definition of modularity matrix is that a product should be designed to satisfy functions after checking that it satisfies constraints.

Moreover, we can define modularity matrices on PCs and FSs for functions and constraints in a way similar to that on DP. The coordinates $\{Y_i\}$ of PC represent the descriptions of products, and the coordinates $\{Z_i\}$ of FS represent company names and brands. Relationships among modularity matrices on DP, PC, and FS are displayed in **Figure 4**, where functions are the same but constraints are different on three base vector spaces.

$$
\begin{tikzcd}
f_i(X) \arrow[r, phantom, "="] & f_i(Y) \arrow[r, phantom, "="] & f_i(Z) \\
DP\{X_j\} \arrow[r, "\Phi 1_{ij}"] \arrow[u, shift left] \arrow[u, leftarrow, shift right, "Gf_{ij}"'] & PC\{Y_j\} \arrow[r, "\Phi 2_{ij}"] \arrow[u, shift left] \arrow[u, leftarrow, shift right, "Gf_{ij}"'] & FS\{Z_j\} \arrow[u, shift left] \arrow[u, leftarrow, shift right, "Gf_{ij}"'] \\
C1_i(X) \arrow[r] \arrow[u, shift left] \arrow[u, leftarrow, shift right, "Gc1_{ij}"'] & C2_i(Y) \arrow[r] \arrow[u, shift left] \arrow[u, leftarrow, shift right, "Gc2_{ij}"'] & C3_i(Z) \arrow[u, shift left] \arrow[u, leftarrow, shift right, "Gc3_{ij}"']
\end{tikzcd}
$$

FIGURE 4: *Relationship among DSMs on DP, PC, and FS*

Here, two mappings are denoted as $\Phi 1_{ij}$: DP→PC and $\Phi 2_{ij}$: PC→FS, which are two examples of DMMs to connect modularity matrices on different base vector spaces *(Danilovic and Browning, 2007; Lindemann et al., 2009).*

For regular matrix $\Phi 1_{ij}$ such that $dY_i = \sum_{j=1}^{n} \Phi 1_{ij} \rightarrow dX_j$, modularity matrices for functions on DP and PC, denoted by Gd_{ij} and Gp_{ij}, are related as $Gp_{ij} = \sum_{k,m=1}^{n} \Phi 1_{ik} \rightarrow Gd_{km} \rightarrow (\Phi 1)^{-1}_{mj}$. In addition, we note that a modularity matrix can be changed by constraints beyond a DMM, because constraints are generally different on DP, PC, and FS.

Regarding the meaning of constraints $C1_i(X)$ in DP, $C2_i(Y)$ in PC, and $C3_i(Z)$ in FS, we can consider that $C1_i(X)$ represents the physical constraints for design, $C2_i(Y)$ represents the equipment constraints for components, and $C3_i(Z)$ represents operation constraints in firms and markets *(Fujimura, 2000).* For example, one operational constraint is the cost of products. Generally, constraints become stronger in order when approaching the market such that there is $\{C1_i(X)\} \supset \{C2_i(Y)\} \supset \{C3_i(Z)\}$ as set inclusion.

Finally, we prove equivalence between the modularity matrices of DSM and AD. The modularity matrix of AD is defined as a mapping ψ_{ij} : FR→PC *(Ulrich, 1995: Suh, 2001)* such that $Y_i = \sum_{j=1}^{n} \psi_{ij} \cdot f_j(X)$.

Since $\psi_{ij}^{-1} = \left(\partial f_i / \partial Y_j\right)$, DSM on PC for functions is

$$Gf_{ij} = \sum_{k,m=1}^{n} \left(\partial f_k / \partial Y_i\right)^{-1} \cdot Kf_{km} \cdot \left(\partial f_m / \partial Y_j\right) = \sum_{k,m=1}^{n} \psi_{ik} \cdot Kf_{km} \cdot \psi_{mj}^{-1}.$$

Since Kfij have only diagonal elements, in terms of modularity, DSM Gfij on PC for functions is equivalent to the modularity matrix ψ_{ij} of AD. Above, we should note that constraints are ignored and that DSM for both functions and constraints on PC may generally be different from the modularity matrix ψ_{ij} of AD.

4. Modularization from Statistical Mechanics

In this section, we introduce a regularization of modularity matrix and entropy from statistical mechanics. From the entropy for modularity matrix, we derive modularizations of DSM and AD such as a real option theory and information axiom. Moreover, we propose a new principle of modularity from the principle of entropy maximization.

From the definition of modularity matrix for functions and constraints, when the initial values are far from the permissible ranges, a modularity matrix on the initial values does not satisfy both functions and constraints. Then, we must engage in trial and error to find initial values satisfying the permissible ranges of both functions and constraints. We improve the definition of a modularity matrix to enable finding the permissible ranges of functions and constraints in order to realize the system although we cannot recognize the solutions in advance or even if the initial values remain far from the permissible ranges. Concretely, we write Gfij and Gcij by using step functions in order to make the permissible ranges explicit. When the parameters α_i in Kfij are between Ai and Bi, denoted as **[Ai,Bi]**, by use of step functions $\theta(\cdot)$, Kfij is more explicitly written as

$$Kf_{ij} = diag(\alpha_i \cdot (\theta(\alpha_i - A_i) - \theta(\alpha_i - B_i))).$$

We introduce a regularization of the modularity matrix by use of a new parameter T to modify the step functions such that

$$\theta(\alpha - A) = \lim_{T \to +0} 1 - \frac{1}{1 + e^{\frac{(\alpha - A)}{T}}}.$$

In **Figure 5**, we draw the probability for distribution functions to realize the system before and after regularization when we set B = −A = 5 and T = 1.

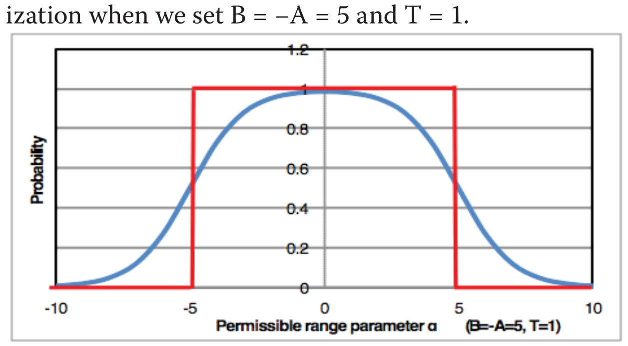

FIGURE 5: *Probability before and after the regularization*

When T is large, finding the solutions from outside the permissible ranges is easier. This new parameter T signifies a capability to find unknown solutions for product design and also a possibility to realize the small performance of product even outside permissible ranges. This regularization is similar to the fuzzy information axiom *(Kulak and Kahraman, 2005).* Concretely, $Kf_{ij} = diag(\alpha_i)$ is regularized as $Kf_{ij}(T) = diag(\alpha_i(T))$

such that

$$\alpha_i \{\theta(\alpha_i - A_i) - \theta(\alpha_i - B_i)\} = \lim_{T \to 0} \alpha_i(T)$$

for

$$\alpha_i(T) = \alpha_i \left\{ \frac{1}{1 + e^{\frac{(\alpha_i - B_i)}{T}}} - \frac{1}{1 + e^{\frac{(\alpha_i - A_i)}{T}}} \right\}.$$

After regularization, the modularity matrices on DP for functions and constraints are defined by replacing Kf_{ij} and Kcij to $Kf_{ij}(T)$ and $Kc_{ij}(T)$, where $Kc_{ij}(T)$ is similarly defined from Kc_{ij}.

Next, we introduce entropy for the modularity matrix from statistical mechanics. The regularized modularity matrix for one function is

$$Gf = \alpha(T) = \alpha \left\{ \frac{1}{1 + e^{\frac{(\alpha - B)}{T}}} - \frac{1}{1 + e^{\frac{(\alpha - A)}{T}}} \right\}$$

for only one module. We note that the regularized modularity matrix for one function is well-defined even when the permissible range of **[A,B]** is limited by a constraint as well as by a function. From a physics viewpoint, the matrix element of a regularized modularity matrix for only one module can be identified as energy for a grand-canonical ensemble system of fermions. From statistical mechanics *(Kubo, 1965)*, a partition function Z is defined as follows:

$$Z = \frac{1 + e^{-\frac{(\alpha - B)}{T}}}{1 + e^{-\frac{(\alpha - A)}{T}}}$$

for a grand-canonical ensemble of fermions. Also, entropy S is introduced as follows:

$$S(\alpha, A, B) = \log\left(\frac{1 + e^{-\frac{(\alpha - B)}{T}}}{1 + e^{-\frac{(\alpha - A)}{T}}} \right) + \frac{\alpha}{T}\left\{ \frac{1}{1 + e^{\frac{(\alpha - B)}{T}}} - \frac{1}{1 + e^{\frac{(\alpha - A)}{T}}} \right\} + \frac{1}{T}\left\{ \frac{A}{1 + e^{\frac{(\alpha - A)}{T}}} - \frac{B}{1 + e^{\frac{(\alpha - B)}{T}}} \right\}.$$

In **Figure 6**, we plot the entropy for the interaction *(B–A)* for T = 1 and α = B/2 by simulation. While in physics, entropy shows the number of states that can be realized in the system, in the field of product design entropy represents the number of possible options in order to realize the system, which may be related to the real option theory *(Baldwin and Clark, 2000).* We conjecture the absolute value of this entropy to represent the value or performance of products to be maximized for product design. The optimal design at the maximum point of entropy may be a concrete example of "dominant design" *(Utterback, 1994).*

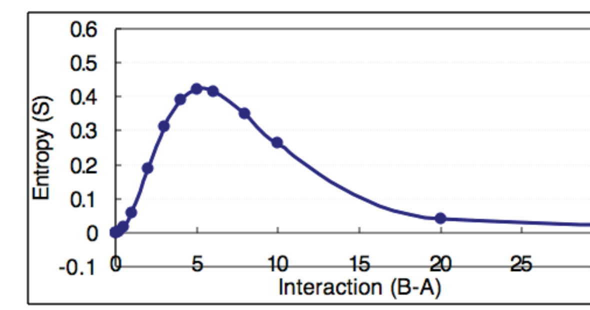

FIGURE 6: *Entropy for interaction*

We verify this conjecture through the modularizations of DSM and AD. We then study two different limits of the entropy for B/T, smaller or larger, drawn as two circles in **Figure 6**. Here, we set A=−B for simplicity. As the first limit of small B/T, where the constraint is strong, the entropy is expanded as

$$S \approx \frac{\alpha B}{2T^2}.$$

Therefore, for small B/T, the entropy is proportional to the size of interaction B. This is logically the same as the modularization of DSM by real option theory *(Baldwin and Clark, 2000)* such that the size of interaction B is similar to the standard deviation σ in real option theory.

As the second limit of large B/T, where the constraint is weak, the entropy is approximated as follows.

$$S \approx -\log\left(\frac{1}{1 + e^{\frac{\alpha - B}{T}}} \right) = -\log \hat{P},$$

where $\hat{P}$ can be interpreted as a probability by which the system satisfies functions without constraints. Therefore, for large B/T, the entropy is approximated to a logarithm of the probability to realize the system. This is logically the same as the modularization of AD by information axiom *(Suh, 2001)*. Finally, regarding applicable cases of DSM and AD, DSM is applicable only to the cases of small B/T, where the constraint is strong, whereas AD is applicable only to the cases of large B/T, where the constraint is weak.

Next, we study the cases of many modules with constraints. When the matrix elements of modularity matrix are rescaled, the modularity matrix for functions is

$$Gf_{ij} = \sum_{k=1}^{n} (\partial f_k / \partial X_i)^{-1} \cdot \alpha_k \cdot [\theta(\alpha_k - A_k) - \theta(\alpha_k - B_k)] \cdot (\partial f_k / \partial X_j)$$

$$= \sum_{k=1}^{n} \alpha \cdot \left[\theta\left(\alpha - (\partial f_k / \partial X_i)^{-1} \cdot A_k \cdot (\partial f_k / \partial X_j)\right) - \theta\left(\alpha - (\partial f_k / \partial X_i)^{-1} \cdot B_k \cdot (\partial f_k / \partial X_j)\right)\right],$$

where we choose $\alpha_k = \alpha$ for all k. Similarly, the modularity matrix for constraints is

$$Gc_{ij} = \sum_{k=1}^{n} \gamma \cdot \left[\theta\left(\gamma - (\partial C_k / \partial X_i)^{-1} \cdot P_k \cdot (\partial C_k / \partial X_j)\right) - \theta\left(\gamma - (\partial C_k / \partial X_i)^{-1} \cdot Q_k \cdot (\partial C_k / \partial X_j)\right)\right],$$

where we choose $\gamma_k = \gamma$ for all k, and the parameters γ_k are between Pk and Qk, denoted as **[Pk,Qk]**. By choice of a measure of delta function, the modularity matrix for both functions and constraints is

$$G_{ij} = \sum_{l=1}^{n} \int \frac{d\gamma}{\gamma} \cdot \delta(\gamma - \alpha) \cdot Gf_{il} \cdot Gc_{lj} = \sum_{l,k,m=1}^{n} \alpha \cdot [\theta(\alpha - A_{iklmj}) - \theta(\alpha - B_{iklmj})],$$

where the intersections are defined as

$$[A_{iklmj}, B_{iklmj}] = [(\partial f_k / \partial X_i)^{-1} \cdot A_k \cdot (\partial f_k / \partial X_l), (\partial f_k / \partial X_i)^{-1} \cdot B_k \cdot (\partial f_k / \partial X_l)]$$

$$\cap\ [(\partial C_m / \partial X_l)^{-1} \cdot P_m \cdot (\partial C_m / \partial X_j), (\partial C_m / \partial X_l)^{-1} \cdot Q_m \cdot (\partial C_m / \partial X_j)].$$

When we identify every matrix element in the modularity matrix as the energy for ground-canonical ensemble of fermions with the same energy α and the same temperature T, entropy for many modules with constraints is written as a sum of entropy, denoted as Sij for every matrix element.

$$S = \sum_{i,j,k,l,m=1}^{n} S_{ij}(\alpha, A_{iklmj}, B_{iklmj}).$$

This entropy represents a value or performance of products with many modules. From the principle of entropy maximization, we propose a new principle of modularity.

"The Principle of Entropy Maximization" states that as approaching the optimal performance of a product, entropy increases.

We study cases of two modules in one system and can write the matrix elements of functions and constraints as

$(\partial f_i / \partial X_j) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $(\partial C_i / \partial X_j) = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, where we assume that ad – bc > 0, ps – qr > 0, a,b,c,d,p,q,r,s ≥ 0. When b = c = q = r = 0, the product design is called "modular." When b ≠ 0 or q ≠ 0, the product design is called "integral." When b = q = 0 and c ≠ 0 or r ≠ 0, the product design is called "hierarchical." For two modules in one system, we write the entropy as

$$S = S(\alpha, A_{11111}, B_{11111}) + S(\alpha, A_{22222}, B_{22222}) + \Delta S,$$

$$[A_{11111}, B_{11111}] = [A_1, B_1] \cap [P_1, Q_1],\quad [A_{22222}, B_{22222}] = [A_2, B_2] \cap [P_2, Q_2],$$

where ΔS is the sum of contributions from the off-diagonal matrix elements of the modularity matrix. Here, we study the contributions of constraints to change modularity. In the case that q ≠ 0, r = 0, and b = c = 0, where the product design is integral,

$$\Delta S = S(\alpha, A_{11112}, B_{11112}) - S(\alpha, A_{11122}, B_{11122}),$$

$$[A_{11112}, B_{11112}] = [A_1, B_1] \cap [(q/p) \cdot P_1, (q/p) \cdot Q_1],$$

$$[A_{11122}, B_{11122}] = [A_1, B_1] \cap [(q/p) \cdot P_2, (q/p) \cdot Q_2].$$

In the case that r ≠ 0, q = 0, and b = c = 0, where the product design is hierarchical,

$$\Delta S = S(\alpha, A_{22221}, B_{22221}) - S(\alpha, A_{22211}, B_{22211}),$$

$$[A_{22221}, B_{22221}] = [A_2, B_2] \cap [(r/s) \cdot P_2, (r/s) \cdot Q_2],$$

$$[A_{22211}, B_{22211}] = [A_2, B_2] \cap [(r/s) \cdot P_1, (r/s) \cdot Q_1].$$

When the permissible ranges of functions are suitably large, modularity can change by the sizes of **[P1,Q1]** and **[P2,Q2]**. We then state the principle of modularity for the sizes of modularity matrix elements. If **[Pi,Qi]** ⊂ **[Pi+1,Qi+1]** for small ranges or **[Pi+1,Qi+1]** ⊂ **[Pi,Qi]** for large ranges, the product design can become hierarchical. If **[Pi+1,Qi+1]** ⊂ **[Pi,Qi]** for small ranges or **[Pi,Qi]** ⊂ **[Pi+1,Qi+1]** for large ranges, the product design can become integral. If the sizes of **[Pi,Qi]** and **[Pi+1,Qi+1]** are similar, the product design can become modular. Therefore, we find that the principle of entropy maximization defines modularity, which we call the principle of modularity. This principle seems similar to the GA-based clustering for weighted DSM by use of the information theoretic method *(Yu et al. 2007)*, which does not include constraints explicitly.

We draw the entropy for two modules with constraints in **Figure 7** when we set α = T = 1 and P1 = –Q1 = P2/2 = –Q2/2, where the sizes of constraints increase in order. In **Figure 7**, the blue line with circles is in the modular case q = r = 0 ; the red line with squares is in the integral case q/p = 1/2 ; and the green line with triangles is in the hierarchical case r/s = 1/2. In **Figure 7**, we see that the value of a product is higher when the product design is hierarchical, in a narrow range of constraints, or more integral in a wide range of constraints. Thus, constraints can change the modularity to increase the value of a product.

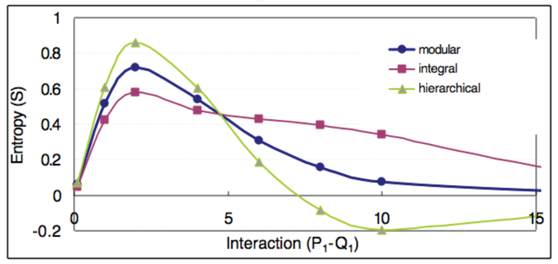

FIGURE 7: *Entropy in modular, integral, and hierarchical cases*

Finally, about the number of modules in one system, we study cases that the number of modules can be optimized in a finite number to maximize the value of product. We consider simple cases to add sets of two modules in one system with integration "ci" as

$$(\partial f / \partial X)_{\{2i+1,2i+2\}} = \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix}$$

, where c/d = 1/2, α = 7, T = 1, Bn = –An = nB1, and B1 = 1, and the other off-diagonal matrix elements of modularity matrix for functions are zero when the constraints are weak and modular. In **Figure 8**, we plot entropy for the number of modules and can see an optimized number of modules in one system. When the number of modules increases beyond the optimized number, it becomes difficult to realize the system without much cost.

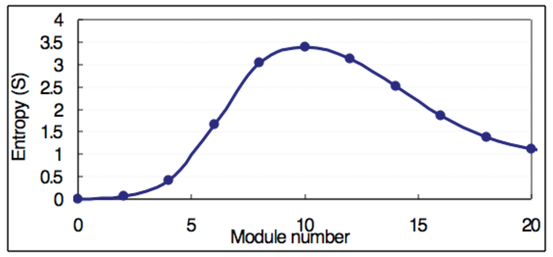

FIGURE 8: *Optimized number of modules*

5. Conclusions

We have demonstrated the equivalence of DSM on PC for functions and AD for modularity matrices and modularization. We have proposed a relationship among three modularity matrices on DP, PC, and FS not only by DMM and MDM but also by showing the differences of constraints. We have regularized a modularity matrix by the new parameter T and have used statistical mechanics to construct entropy for fermions. In addition, we proposed a new principle of modularity from the principle of entropy maximization, which determines how modularity should change in order to realize the highest value of products.

Future scholarship might consider applying our theory to the assembly and process industry. Although our model addresses the regularizations of a modularity matrix by statistical mechanics for fermions, we anticipate that a similar approach of physics will also be useful for studying the optimal designs of products.

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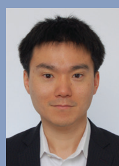

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