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EARNED COST COMPLEMENTS EARNED SCHEDULE, GENERATES AN ESTIMATE OF THE FINAL COST, AND FORMALLY UNIFIES EARNED VALUE MANAGEMENT

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ABSTRACT: This research defines a new quantity called earned cost, and, analogous to the manner in which earned schedule predicts a project's final duration, earned cost leads to an estimate of the final cost. The earned-cost cost estimate is proven to be identical to the traditional cost estimate at completion. It is proven that the earned schedule-based duration estimate and the earned cost-based cost estimate formulas are universal: they apply at all project stages and to all projects. Although the formal proof of universality provides little project management insight, a graphical method clearly shows the universality of the formulas. Using the graphical method, the paper also shows that the duration estimate determined from the critical path is, in fact, formally identical to that obtained from earned schedule. Factors that determine the accuracy of the estimates of the final duration and cost are defined. The result is a formal unification of Earned Schedule, Earned Cost, and Critical Path into Earned Value Management. This confirmation reassures project managers, who know now that they may use any of the "earned" methods on any project with the confidence that they will obtain timely and reliable estimates for the final duration and cost.

Keywords: Earned Value Management (EVM), Earned Schedule, Earned Cost, Critical Path Method (CPM), Project Duration Forecasting, Schedule Estimation, Cost Estimation, Cost and Duration Forecasting Accuracy, Earned Duration Management.

1. Introduction

Two typical definitions of a project are, "A temporary endeavor undertaken to create a unique product, service or result" (PMI, 2021) and "every project is unique in what it delivers (and) no two projects (are) the same" (Bennett, Buttrick, & Stanton, 2018). Uniqueness, then, is an essential attribute of a project. If each project is unique, why should one expect a standard set of methods to result in effective predictions for all projects? Proposed cost and duration estimation methods are claimed to be accurate for all projects, thus establishing a basic tenet of project management: that one can predict the end state of a project from current data. Earned Value Management (EVM) arose in the Department of Defense in the late 1960s as the Cost/Schedule Control Systems Criteria policy. EVM has been shown to provide accurate and timely estimates of the final cost (Fleming & Koppelman, 2010). Lipke (2003) extended EVM by defining the Earned Schedule (ES), which was the basis for a method that, for the first time, provided an accurate and timely estimate of the final duration. Research has validated that such methods are effective over a wide range of both real-world and simulated projects (Vanhoucke, 2012; Vanhoucke & Vandevoorde, 2007).

While the standard methods are undeniably effective in practice, how can one justify applying standard estimation methods to unique projects? Do projects have fundamental structural characteristics that guarantee that the estimation methods are generally effective? This paper completes the foundational theory begun with EVM by proving that it is legitimate to apply the standard estimation methods to any project.

At any time, t, the ES method provides an estimate of the project's final duration, $T_f(t)$:

$$T_f\left(t\right) = \frac{tT_p}{E_s(t)} \qquad (1)$$

where, $Es(t)$, is the earned schedule at time t, and T_p is the planned duration; see Figure 1 (Lipke, 2003).

By analogy with the earned schedule, we define a new quantity called the earned cost, Ec (t) , as the time of the intersection from the earned value data point at time t to the actual cost data; see Figure 1. Given the planned cost, C_p (i.e., the budget), and following the same derivation method as for equation 1, we derive a new formula for the estimate of the final cost:

$$C_f\left(t\right) = \frac{tC_p}{E_c(t)} \qquad (2)$$

The similarity between the duration and cost estimation formulas, equations 1 and 2, is striking.

However, equation 2 only applies to projects with a constant cost rate. Therefore, we also derive a completely general cost estimation formula that applies to all projects, even those with S-shaped (planned, earned, and actual) cost expenditures. We proved that this cost estimation formula, derived from the earned cost, is identical to the traditional Cost Estimate at Completion (CEAC), which is defined as the ratio of the budget to the cost performance index, *EAC* (*t*) = *BAC*/*CPI* (*t*) (PMI, 2021).

Our second research contribution is to prove that both the duration and cost estimation formulas are universal, i.e., they explicitly apply at all project times and to all projects. That is, no matter the stage of the project nor the shape of a project's planned, earned, and actual cost profiles, the estimation formulas provide valid cost and duration estimates.

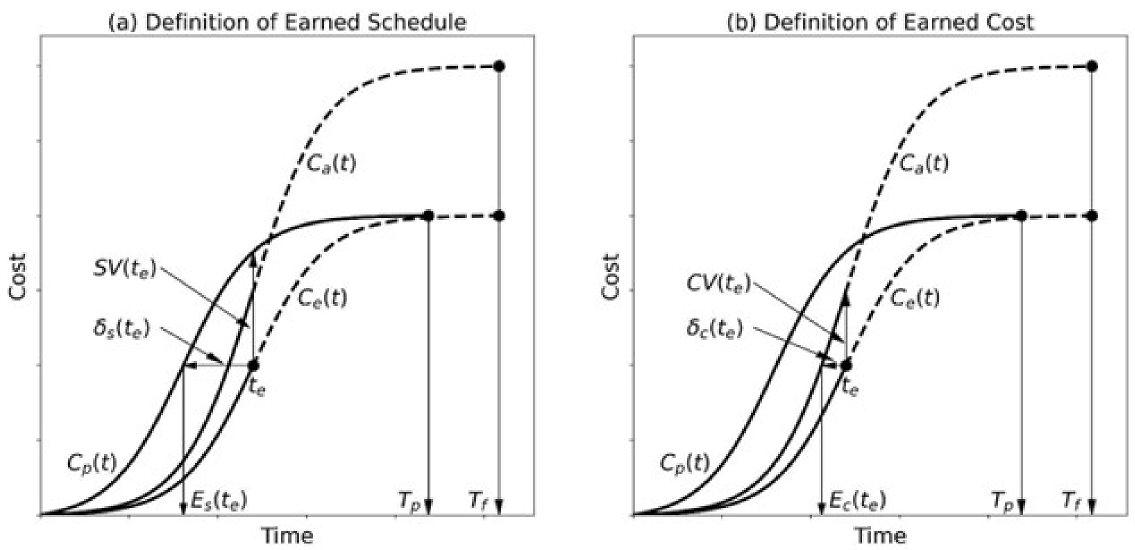

Figure 1: Using the three Earned Value Quantities — Planned Cost, C_p (*t*), Earned Cost, C_e (*t*), and Actual Cost, C_a (*t*) — at Any Time, t = t_e, Graph (a) Shows Definitions of Earned Schedule, E_s (t_e), Schedule Variance, *SV* (t_s), and Schedule Delay, δ_s (t_e); Graph (b) Shows Definitions of Earned Cost, E_c (t_e), Cost Variance, *CV*(t_e), and Cost Delay, δ_c (t_e).

Because the earned cost estimate is identical to the traditional estimate at completion (CEAC), the traditional CEAC inherits all the properties established for the earned cost approach, including universality. These results are valuable to project managers who can confidently estimate costs and durations, whatever their project. These same research contributions should also be valuable to researchers. One can imagine a situation where the estimation formulas were proven to apply only to limited sets of project cost profiles. This is not an abstract issue as that situation arose when it was found in a prior study; the duration estimation formula did not apply to projects with cost profiles that closely followed the Cioffi profile. We resolved that situation by proving the cost and duration estimation formulas are valid for all projects. Since we use only standard EVM methods, the results from previous research studies stand without change. Thus, this research enhances the value of the duration and cost estimation both theoretically and practically.

Kim and Kwak (2018) showed that conducting an early warning forecast to detect potential cost and schedule overruns is essential for timely and effective decision-making and that even the best forecasting model may change as the project progresses. The third research contribution of this paper addresses this issue by defining and estimating the accuracy of the estimation formulas as the project progresses. The theory allows us to explain the characteristics of estimation errors typically found in practice (Mamghaderi, Khamooshi, & Kwak, 2021). Further, we demonstrate, using a simulation, that the accuracy of the new cost estimation formula is comparable to that of the duration estimation formula as well as to that of the traditional EAC formula. The formal proof of the universality of the estimation formulas is mathematical and provides little project management insight as to why it is true. Therefore, the fourth research contribution is the presentation of a graphical method that explains why the final cost and duration formulas are universal.

There appear to be two independent methods of determining the final project duration: the Earned Schedule Method (ESM) and the Critical Path Method (CPM) (Clark, 1922). CPM is often described as an independent duration estimation method as it uses only the critical subset of project activities while ESM uses all available planned and earned values. The final research contribution is to use the graphical method to prove that the ESM and CPM methods are, in fact, formally equivalent, thus unifying what were previously thought to be different approaches to duration estimation. These results unify the critical path method with the earned schedule and earned cost methods into an integrated and universal Earned Value Management. This research is based on the standard definitions of EVM and ES (PMI, 2011, 2021). No approximations are required, and the results are, therefore, exact. Also, since only standard EVM definitions and terminology are employed, all research on EVM remains valid and applicable.

The paper is structured as follows: First, the theoretical and practical background for the research is discussed. The earned cost is defined and the general formula for the estimate of the final cost is derived. We then prove that both the earned schedule and earned cost estimation formulas are universal and describe a graphical method that explains why the estimation formulas are universal. Using this graphical method, the universal ES-based duration estimation formula is formally proven to be equivalent to the duration estimate obtained from CPM. Finally, the characteristics of the estimation errors are explained. The paper concludes with a discussion of the theoretical and practical research findings, their practical implications, and avenues for future research.

1.1. Historical Development of Cost and Schedule Estimation Methods

Parr (2006) made an early attempt to relate a project's observable management parameters (e.g., cost and schedule) to its internal structure. He assumed that activities were linked by finish-to-start constraints and that since there were fewer activities towards the end of the project, activities were less likely to have descendants as the project progressed. The resulting S-shaped curve provided a theoretical understanding of the relationship between the project's internal structure (network dependencies) and its observable, management parameters (cost and schedule). For real-world projects, however, it was hard to estimate the internal parameters such as the number of descendant activities over time, and the Parr curve fell out of favor.

Parr also provided a theoretical derivation of the Putnam-Norden-Rayleigh (PNR) (Putnam, 1978) curve by assuming that the cost rate is proportional jointly to the workers' learning skill and the number of activities left to be completed (Putnam, 1978). Parr showed that a learning skill linear in time resulted in the PNR curve, which has significant realism and tractable mathematics, without too much complexity (Boehm, 1981). Fleming and Koppelman (2010) enhanced Earned Value Management (EVM), which can provide accurate cost estimates, but only indicates schedule progression. For example, while a specific schedule performance index, e.g., SPI = 0.8, indicates the project is behind schedule, the value of the SPI does not generally lead to an estimate of the delay. To correct this deficiency, Lipke (2003) defined the Earned Schedule (ES) and provided the missing schedule estimation formula (Lipke, 2009).

However, Lipke's duration estimate relied on the assumption of a constant effort rate. Ward and Litchfield (1980) stressed that projects continually change, which invalidates the linearity assumption. Prior literature added that the linearity assumption causes problems when short-term schedule indices are used to forecast long completion times. Prior literature also described the schedule estimation formula as only applicable to linear cost profiles. Vanhoucke (2012) confirmed those ideas in practice by showing that the network topology is a significant factor in variability. According to prior literature, the sensitivity of forecasting accuracy to activity information and concluded that failures occur when parallel, non-critical-path activities generate incorrect schedule warnings. These studies emphasize the need for general duration and cost estimation techniques that apply to S-shaped labor profiles.

A new approach, called Earned Duration (ED). ED is based on the same EVM approach, but only uses activity dates and durations and, by eschewing the use of cost data, decouples the cost and schedule forecasts. The ED method was refined in a way the time dependency and substituted the properties of the network activities. Mamghaderi et al. (2021) demonstrated, with detailed simulations, that the duration forecasting accuracy of the ED method was at least as good as, and often superior to, the ES method. A similar conclusion is determined in a prior study that studied 57 projects from different sectors. An other prior study provided a useful introduction to ED and provided an excellent review of duration forecasting using both ES and ED methods. They also proposed an alternative to earned value, called

EV^d , that provides a simpler and more consistent determination of the activities' earned durations, enhanced visibility of potential schedule problems, and appears to be a viable alternative to ES.

Despite the linear restriction and, presumably, since it was the only approach available, the ES-based estimation formula was shown to accurately predict the project schedule for a wide variety of real-world and synthetic projects (Vanhoucke & Vandevoorde, 2007). Using statistical prediction and testing methods, a prior study showed that the ES method worked well. A prior study proposed a framework to improve the predictive power of the Planned Value-based forecasting technique (Anbari, 2003). Using a large dataset that they made publicly available, a prior study compared the accuracy and timeliness of EVM, ES, and ED and concluded that all methods of schedule forecasting are good enough for practical work, but that the accuracy of ED was generally slightly superior.

In projects with multiple parallel activity paths and especially when durations vary significantly, earned duration metrics sometimes fail to provide reliable duration monitoring and forecasting (Wood, 2018). Using only critical-path activities helps to overcome distortions caused by non-critical-path activities, and enhances forecast accuracy, which depends strongly on the topological features of the project network (Valadares Tavares, Antunes Ferreira, & Silva Coelho, 1999, 2002). Simulation models demonstrate that the serial or parallel nature of the network affects duration forecasting (Vanhoucke, 2012; Vanhoucke & Vandevoorde, 2007). Wendell, Lowe and Gordon (2022) also observe that earned value metrics for assessing project schedules suffer from the same confounding impact of parallel activities. The theory presented here allows us to make progress on the reason that parallelism impacts forecasts. Forecasting effectiveness has been discussed in different industries: construction (Rujirayanyong, 2009; Ward & Litchfield, 1980), hydroelectric plants (Urgilés, Claver, & Sebastián, 2019), and defense projects (Cho & Lim, 2018). A prior study determined the evolution of project cost estimates during the preconstruction phase of 29 Dutch flood defense projects using a case study approach.

According to prior literature, researchers also removed the linearity constraint by rigorously defining a general theoretical foundation for ES and showed how to derive the schedule estimation formula even for projects with non-linear cost profiles. They also found that Lipke's linear schedule estimation formula applied to several non-linear labor profiles, but also reported an exception: schedule estimates were incorrect for projects that followed the Cioffi profile. It is proposed that the solution to a differential equation frequently used in ecology can successfully reproduce project cost curves (Cioffi, 2006). This paper resolves that issue by demonstrating how to write the Cioffi profile and, indeed, any cost profile so that the duration estimation formula applies and accurately predicts the schedule.

Prior literature showed that the budgeted cost and planned duration provided a reasonable basis for estimating the final cost and duration. A prior study proposed a linear model that improved the EVM forecasting accuracy by an average of 13% when applied to 131 different projects. A prior study analyzed the Time Duration Method (TDM). A prior study proposed a method to quantify the duration forecasting accuracy. Because readily available algorithms generate reasonable accuracy, many regression techniques have been proposed. A prior study used stochastic S-curves to model the variability of construction costs and duration and realized additional benefits from comprehensive project control. Lee (2005) used simulation to overcome the randomness in the activities' schedule.

Many authors have extended the EVM method, and Savklint and Danielsson (2021) provide a useful, detailed summary. The exponential smoothing technique has been shown to improve final duration forecasts. A prior study found improved forecasting by using a Kalman Filter forecasting method. In a slightly different approach, a prior study evaluated ED performance indices for project duration forecasting. De Marco, Ottaviani and Bolognesi (2024) proposed a framework to deal with inflation by estimating costs based on trend and seasonality analysis through the Holt–Winters method.

Several studies used complex statistical analyses. Ko and Cheng (2007) focused on the prediction of project success using a model based on neural networks, genetic algorithms for optimization, and fuzzy logic for approximate reasoning. The same technique was later used to capture the cost profile's S-shape. A prior study proved that nonlinear time estimates give a better indication of the total duration because the logistic equation can overcome the SPI bias at the end of the project, where *SPI → 1.*

The Gompertz cost profile and nonlinear regression predict the cost. Huynh et al. (2020) validated that model using data from 25 real-life projects. A prior study proposed a multivariate model that led to performance improvements and practical advantages over traditional approaches. Qiao, Labi and Fricker (2019) developed a two-stage survival analysis that combined nonparametric estimation with a parametric time-to-event model that related duration to external covariates. A prior study described about a neural network model to estimate the duration and demonstrated its superiority to standard approaches. Sackey, Lee and Kim (2020) developed a duration estimation model which removed cost as a proxy but relied on linear regression analysis. A priror study described the schedule overruns by fitting predefined distributions to project data and exploited risk data to increase duration forecast accuracy. Warburton, De Marco and Sciuto (2017) demonstrated that accurate schedule estimates can be made by using nonlinear labor profile fitting, especially in the early stages of project development when the practical benefits are the greatest for project teams to take their corrective actions.

Other techniques have addressed the need for a practical and straightforward approach to forecasting the schedule. In prior literature, performance stability and change-point analysis are to compute performance indexes that improved schedule forecasts. A prior study proposed and tested the to-complete-schedule-compression-ratio (TCSCR), which was used to detect false warnings of schedule delays and to correct the schedule forecast. Dehghan, Mortaheb and Fathalizadeh (2020) developed a Heuristic Estimation Method and tested it on selected projects where the prediction accuracy reached 95%. While these practical studies establish the undoubted effectiveness of EVM and ES, there remains no explanation for why they should work so well. This paper establishes a theory that explains the effectiveness of EVM and ES.

2. Research Methods

We begin by defining the concept of the cost profile and reviewing the derivation of the formula for the project's duration estimate at completion based on the earned schedule (ES). We then define the earned cost (EC) and follow the same process as for ES to derive a new formula for the project's cost estimate at completion. We then prove that both the earned-schedule-based duration estimation formula and the earned-cost-based cost estimation formula are universal, i.e., they work at all project times and for all projects. Since the formal derivation provides little practical insight about why the formulas are universal, we present a graphical explanation of why the estimation formulas apply to all projects. Using this method, we also prove that the universal ES-based duration estimation formula is formally equivalent to the duration estimate obtained from the CP method and discuss the implications of this result.

Finally, the theory allows us to define the types of estimation errors that occur, and we derive an estimate of the impact of the different types of errors on the duration estimate. We then demonstrate the application of these techniques using a simulation, which confirms the theory and explains the general behavior of the accuracy of estimates over the life of the project.

2.1. Cost Profiles

We define the "cost profile" as a description that yields the cumulative effort (whether planned, earned, or actual) as a function of the time elapsed from the beginning of the project. The effort includes the costs of all resources, such as labor and equipment, and we use the three classic EVM cost profiles: the planned value, the earned value, and the actual cost. While monetary units are frequently used for effort, such units are not required to use EVM. The Practice Standard for EVM emphasizes that performance is measured by the completion of "specific and tangible" attributes and that "observable evidence of a tangible product or progress is required." (PMI, 2011) The definition of earned value considers activities to be complete upon measurement of observable project parameters. While we use the term "cost profile," we explicitly acknowledge that the presented methods apply to whatever measure of performance is appropriate to a specific project.

2.2. Earned Schedule Estimation Formula

The cumulative planned, earned, and actual profiles are functions of time: C_p (*t*), C_e (*t*), and C_a (*t*). The planned project ends at time $t = T_p$, and the actual final project end is at $t = T_f$, so if $T_f > T_p$, the project is delayed. The schedule delay, δ_s (*t*), is defined (see Figure 1) as the time interval from the current earned value to the horizontal intersection with the planned value, at $t - \delta_s$ (*t*),

$$C_e(t) = C_p[t - \delta_s(t)] \quad (3)$$

The earned schedule, E_s (*t*), is therefore, the time from the start of the project to that of the above intersection;

see Figure 1:

$$E_s(t) = t - \delta_s(t) \quad (4)$$

The budget, B, is the total planned cost, and at the end of the project, if all work is completed, the total earned value equals the total planned value: $B = C_p(T_p) = C_e(T_f)$ (PMI, 2011).

We first consider profiles where the three "C" quantities vary linearly with time:

$$C_p(t) = B\left(\frac{t}{T_p}\right), \quad C_e(t) = B\left(\frac{t}{T_f}\right), \quad C_a(t) = C_f\left(\frac{t}{T_f}\right) \quad (5)$$

Where C_f is the (actual) final cost. Applying the delay, equation 3, and using the definition of Earned Schedule, equation 4, gives,

$$B\left(\frac{t}{T_f}\right) = B\left(\frac{t - \delta_s(t)}{T_p}\right) \quad \Rightarrow \quad T_f(t) = \frac{t\ T_p}{E_s(t)} \quad (6)$$

In practice, the final duration is unknown and to be estimated, and we denote the duration estimate computed at time t as $T_f(t)$ to distinguish it from the actual final duration, T_f.

Equation 6 is a fundamental relation: at any project time t, it computes a final duration in terms of the earned schedule. If the planned and earned value project data exactly follow the planned and earned profiles, equation 6 perfectly predicts the value of the final duration at any time.

Lipke also defined a time-dependent Schedule Performance Index, SPI_t, as

$$SPI_t = \frac{E_s(t)}{t} \quad (7)$$

Substituting this in equation 6 leads to,

$$T_f(t) = \frac{T_p}{SPI_t} \quad (8)$$

These are the linear earned schedule duration estimation formulas. In the appendix, we prove that these formulas apply to all projects, even those with nonlinear cost profiles. In that sense, the duration estimation formulas are universal.

3.3. Earned Cost Estimation Formula

In analogy with the earned schedule, we define the earned cost as,

$$C_e(t) = C_a[t - \delta_c(t)] \quad (9)$$

Where $\delta_c(t)$ is the horizontal cost delay from the earned value at time t to the intersection with the actual cost; see Figure 1. We again consider the case of linear cost profiles defined in equation 5 and apply them to the definition of the earned cost, giving,

$$B\left(\frac{t}{T_f}\right) = C_f\left(\frac{t - \delta_c(t)}{T_f}\right) \quad (10)$$

The goal is to estimate the project's final cost at time, t, which we denote as, $C_f(t)$, to distinguish it from the actual final cost, C_f. Defining the earned cost as $Ec = t\ \delta c(t)$, equation 10 becomes,

$$C_f(t) = \frac{t\ B}{E_c(t)} \quad (11)$$

By analogy with the earned schedule, we define a quantity, CPI_t, as

$$CPI_t = \frac{E_c(t)}{t}$$

which leads to,

$$C_f(t) = \frac{B}{CPI_t} \quad (12)$$

As noted above, the similarity of the earned cost estimation formulas to the earned schedule estimation formulas is noteworthy. The above cost estimation formulas are new and distinct from the well-known cost estimate at completion, defined as the budget divided by the cost performance index,

$$CEAC(t) = \frac{B}{CPI(t)} \quad (13)$$

The above analysis applies to the case where the cost profiles are linear. In the appendix, we derive the general earned cost-based cost estimation formula and prove that it is identical to the traditional cost estimate at completion, *CEAC* (t), equation 13. In the Appendix, we also prove that the above earned cost-based estimation formulas are valid for all project times and for all projects, i.e., they are universal. While the use of equation 13 is ubiquitous in practice, these results prove the equation is valid for all projects.

3. The Rationale for Universality

The universality of the earned schedule and earned cost estimation formulas poses an interesting research question. Why should these formulas apply to all project cost profiles, from the simplest linear cost profile, in which the cost rate is constant, to complex S-shaped cost profiles, with extensive parallelism in the activities, that are typical of large projects? While the derivations of the estimation formulas are formally correct, the analysis provides little project management insight. Therefore, we explain universality in practical terms. Consider the project shown in Figure 2. The earned schedule method of estimating the duration is shown in subplot (d), where the *cumulative* planned and earned cost data (dots) follow S-shaped profiles. The schedule delay, $\delta_s(t)$, is determined at time $t = 9$ by drawing the horizontal line from the earned value data point back to the intersection with the planned value data. That occurs near the time $t = 5$, and the intersection determines the earned schedule, $E_s(t)$.

Now consider the subplots (a) and (b), which show the cost *rate* profiles, planned (a) and earned (b), in each time period. Consider the activities in subplot (a), which were planned to be completed at time $t = 5$. Those activities were delayed and actually completed at time $t = 9$, which is simply an application of the definition of earned value. The planned activities, indicated in Figure 2(a) with dark gray shading, are simply those at the bottom of each time interval. The earned activities indicated in Figure 2(b) with gray shading are simply the activities at the bottom of each time interval.

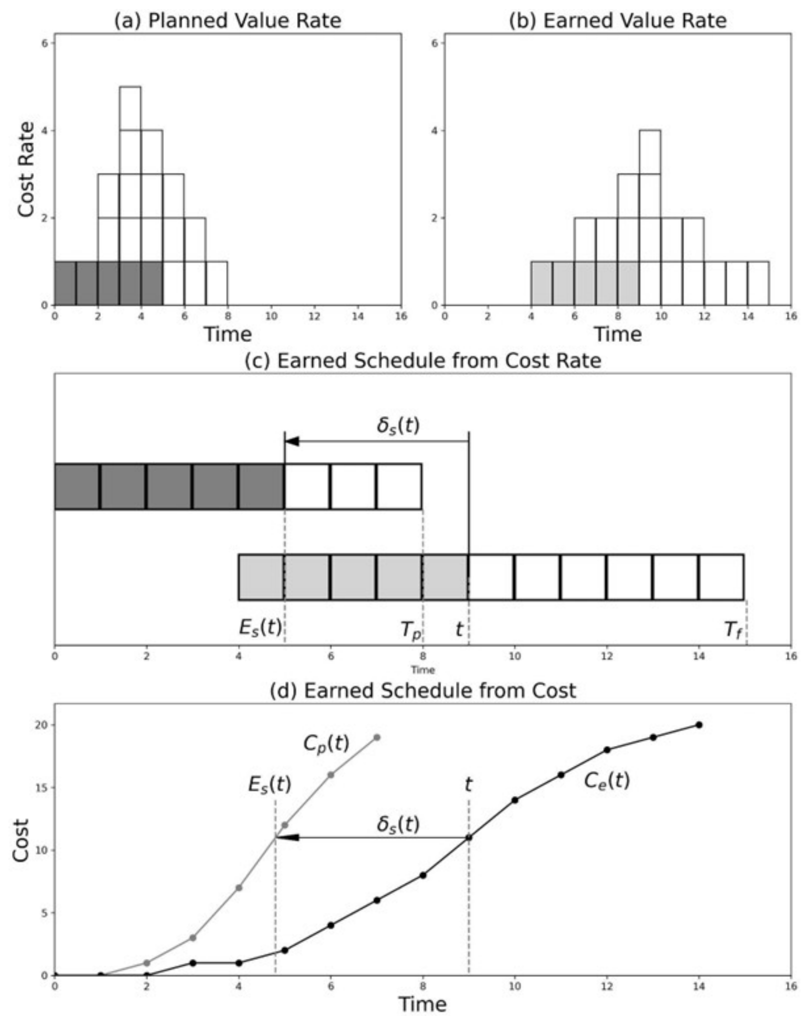

Figure 2: Planned (a) and Earned (b) Cost Rate Profiles. The Earned Schedule, $E_s(t)$, Can be Obtained from both the Rate Profiles (c) and the Cumulative Profiles (d).

In subplot (c), for each time interval, only the planned and earned activities at the bottom of those intervals are plotted and placed at the correct times, so that they are on the same timescale as the cumulative subplot (d). The top row of subplot (c) shows a single activity (the bottom one in the interval) for each *planned* cost rate interval, while the bottom row shows a single activity for each *earned* cost rate interval. The activity at the bottom of the earned rate time, $t = 9$, was completed: the rightmost activity shaded gray in the lower row. This activity was planned to be complete at time, $t = 5$: the rightmost activity shaded black in the planned cost rate (top row). The time delay in subplot (c) from the earned value activity back to the planned activity is the delay, $\delta_s(t)$. The figure shows that this delay is exactly the same as the delay depicted in subplot (d), $\delta_s(t)$, i.e., the schedule delay. Further, the intersection time with the planned cost profile is the earned schedule, E_s (t), and this duration is also exactly the same as the value of E_s (t) depicted in subplot (d).

To estimate the schedule progress, an estimate of E_s (t) is required, and Figure 2 shows that the value of E_s (t), from either the cost rate (subplot (c)) or the more traditional cumulative cost (subplot (d)), can be used. The cost rate value of E_s (t) in Figure 2(c) only required one activity, but all activities in the interval have the same earned value definition: they were all planned to be completed at t = 5, were delayed, and actually completed at t = 9. The bottom activity in the interval was selected to make the diagram clear. The key point is that only one activity from each of the planned and earned value intervals is required to estimate the earned schedule: the number of activities in the interval does not affect the estimate of E_s (t). This is the practical explanation of the result that the estimation of the duration is independent of the shape of the cost profile: neither the total number of activities at the planned time, t = 5, nor those at the earned time, t = 9, affect the delay calculation. The same analysis applies to earned cost since the definition of earned cost is identical to that of earned schedule. Therefore, the cost estimation formula is also independent of any specific cost profile.

EVM does not actually require the identification of which particular activities were completed at t = 9. In practice, activities planned for completion at t = 3 or 4 or 5 may be completed at t = 9. For example, activities scheduled at t = 3 may be delayed longer than others at that time and be completed at t = 9. Or, activities at t = 5 may be delayed less than others at that time and be completed at t = 9. All activities are accounted for at the end of the project because all planned activities earn value when they are completed. While using subplot (c) is theoretically equivalent to subplot (d), subplot (c) only uses a single activity, which makes the resulting duration estimate subject to unpredictable and atypical schedule variations. It is suggested, therefore, that the cumulative calculation illustrated in subplot (d) is more likely to yield a more accurate duration estimate because it averages over more data. Nevertheless, the key theoretical result holds: the duration and cost estimation formulas do not depend on the cost profile, i.e., the estimation formulas do not depend on the number of activities at each time. The duration and cost estimation formulas are formally proven to be universal; they apply at all project stages and to all projects.

4. Critical Path

When using the critical path method, the duration is determined only by the subset of activities on the critical path. In contrast, Figures 1 and 2 show that the ES method of determining the duration uses cumulative data, i.e., all available data. Now consider Figure 3. Suppose the activities that were planned to be completed at t = 2 were delayed and completed at t = 4. One draws the arrow (black) from the earned value data point to calculate the earned schedule, E_s (t). One should note, however, that, by definition, all activities at t = 4 are the same distance (i.e., same time delay) from the corresponding activities scheduled for completion at t = 2. Therefore, all activities at t = 4 contain the same, complete set of information about E_s (t).

In particular, the shaded activity (gray) in Figure 3 contains all information about E_s (t). However, there is nothing special about the shaded activity, any activity in the interval at t = 4 could have been selected. Therefore, a special activity can be selected: the one on the critical path (CP). The gray (CP) arrow can be drawn to determine E_s (t). Therefore, the critical path activity in any time interval can be used to determine the earned schedule and generate an estimate for the final duration. Thus, the duration determined from the critical path method is identical to that estimated from the earned schedule method.

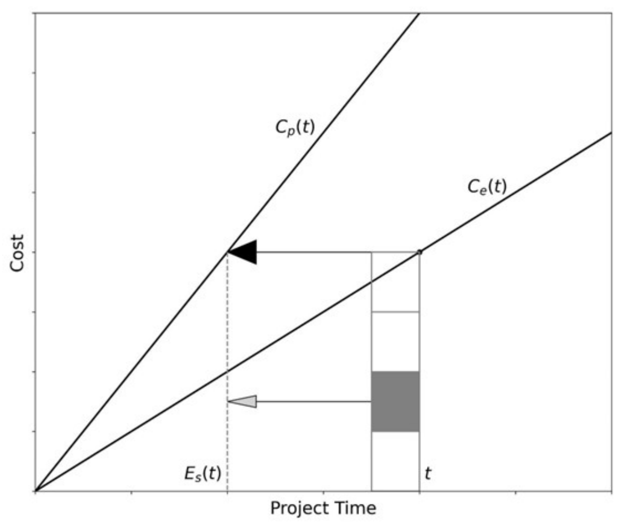

Figure 3: The Relation between the Earned Schedule, E_s (t), and the critical path. The shaded square denotes the critical activity at t = 4.

4.1. Estimation Accuracy

Equations 3 and 9 are more than definitions; they are functional relations between the earned value and the planned value, and the earned value and the actual cost. They are valid over the entire project duration. That the curves are mathematically related for all times means that the duration and cost estimates are perfectly accurate when the deviations between the project data and the cost profile are zero. When the project data fall exactly on the cost profiles, the duration and cost predictions was perfectly accurate. Therefore, the accuracy of the estimates is determined by the deviations between the real-world project data and the cost profiles.

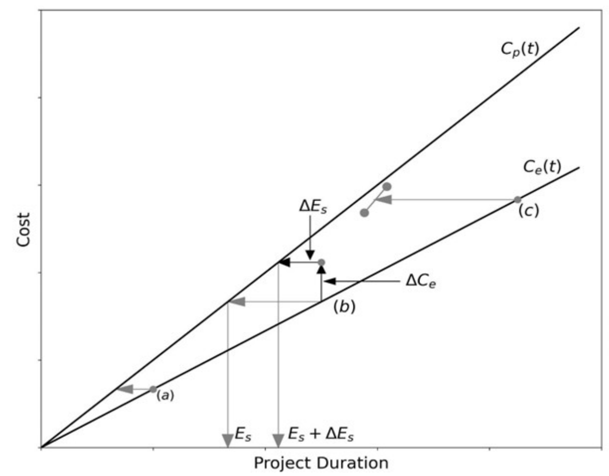

Figure 4: Types of Schedule Estimation Errors.

Since actual project data do not exactly follow planned, earned, and actual cost profiles, it is useful to analyze how prediction accuracy depends on the deviations between the real-world project data and the cost profiles. The impact on the duration estimate of different types of deviations are illustrated in Figure 4. For case (a), the earned value data point (gray dot) falls exactly on the earned value cost profile, so the duration estimate for this time was perfectly accurate, having zero error. Since errors occur when the data do not fall on the cost profile, two types of errors can be identified and these are shown in Figure 4, cases (b) and (c).

4.2. Case (b): Deviation Errors

In this case, there is a deviation, Δc_e, between the earned value data (gray dot, point (b)) and the earned value cost profile, $C_e(t)$. To calculate the earned schedule, a line is drawn horizontally from the data to the intersection with the planned cost curve, $C_p(t)$. Here, the earned value data point's value is greater than the earned cost profile and, as a result, the earned schedule, E_s (t), increases to E_s (t) + ΔE_s (t). ($C_p(t)$) is generally monotonically increasing because project effort is rarely negative.) Since, E_s (t) increases and is in the denominator of the duration estimation equation, the duration estimate decreased. Therefore, error in duration is opposite in sign to the deviation, ΔC_e.

4.3. Case (c): Interpolation Errors:

In this case, the error is due to the interpolation between the planned value data points. The two gray dots represent planned value data points, and the standard ES calculation linearly interpolates between them. If the resulting intersection with the horizontal projection from the earned value data point does not fall on the planned value profile, an error results in the duration estimate. At any time, both types of errors may occur simultaneously. However, interpolation errors, case (c), can be reduced as follows. One can fit the planned data to a suitable cost profile curve such that the planned curve is zero at t = 0 and Cp (Tp) = B. If the fit is reasonable, the residuals were randomly distributed, and the fitted curve average out the deviations. Using the fitted curve for the project's planned data should improve duration estimation accuracy by reducing the interpolation errors.

4.5. Data Analysis: Estimating Deviation Errors

Deviation errors are common because they arise from variations in the completion times of project activities and their impact can be estimated as follows: If the deviation between the earned value data and the earned cost profile is ΔC_e, the fractional error in the final duration is:

$$\frac{\Delta T_f}{T_f(t)} \approx -\frac{\Delta C_e}{C_e(t)} \quad (14)$$

The derivation of this result is given in the Appendix. This result is useful because it explains several well-known, characteristic features of duration estimation errors. Activities are often delivered late, which results in a lower earned value, which in turn results in a negative deviation. The minus sign in equation 14 predicts that a negative deviation resulted in a positive error in the estimate of the final duration. This is as expected because the late delivery of activities reflects an increase the project's estimated duration. Equation 14 also explains the general behavior of the accuracy of duration estimates over the life of the project. Early on, when few activities have been completed, C_e (t) is small and, because it is in the denominator of Equation 14, the effect of the deviation is magnified. On the other hand, as C_e (t) increases, the deviations decline relative to the growing earned value data and the error decreases, i.e., the accuracy increases. Both effects are illustrated in Figure 5. In Figure 5(a), significant random noise was added to the project's earned and actual cost rate profiles (smooth curves). The cumulative cost profiles were then computed (see Figure 5(b)) and one can observe that even significant randomness is smoothed out, reducing the deviations. Figure 5(c) shows the estimate of the final duration, which was computed using the earned schedule method, designated as $T_f(t)$, and the final cost, which was computed using both the earned cost method, $C_f(t)$, and the cost estimate at completion method, *CEAC* (*t*). Figure 5(d) shows the percent errors for each of the estimates. The errors are averaged over intervals of 10% of the project duration using the standard mean absolute percent error (MAPE) technique:

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{T_f - T_f(t)}{T_f}\right| \quad (15)$$

Figure 5(d) clearly shows the agreement between the declining pattern of the errors and the theoretical prediction of eq. 14. The figure also demonstrates that the accuracy of all three estimation methods is comparable over the life of the project. Thus, a project manager can use whichever method is most appropriate. The estimation accuracy remains excellent even when considerable deviations occur in the cost rate profiles (i.e., week to week staffing).

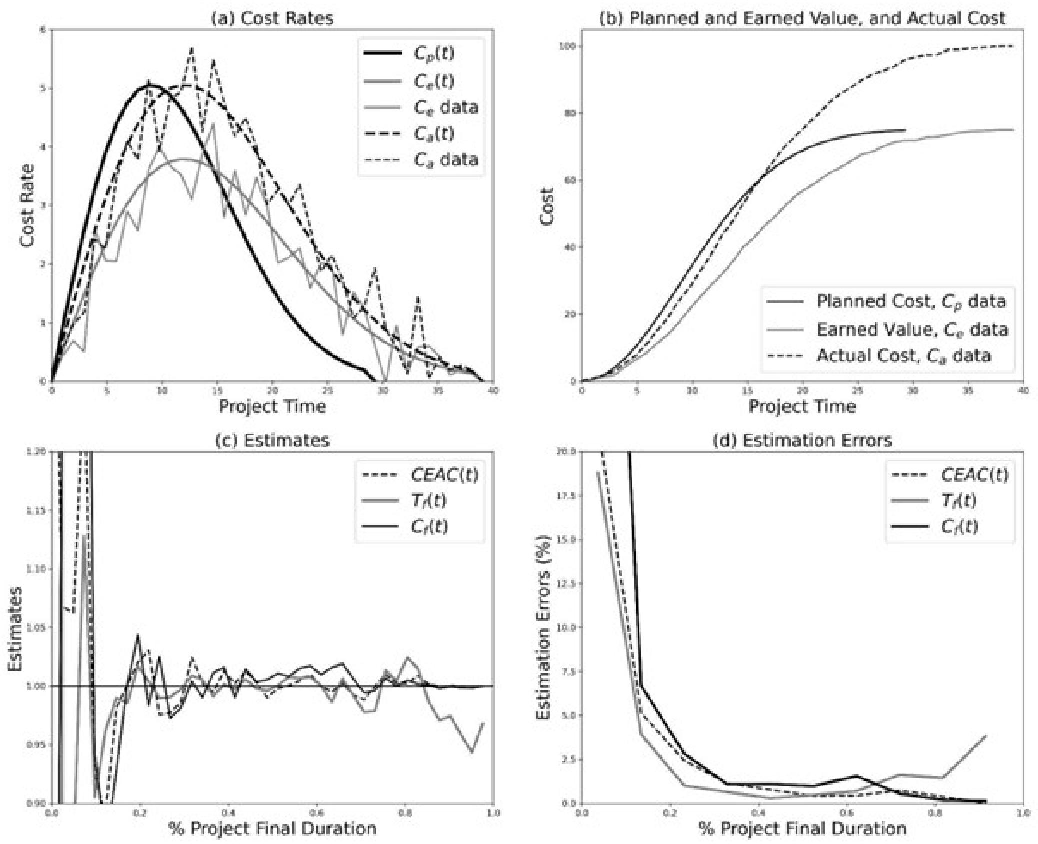

Figure 5: Estimation Errors. Earned and Actual Effort Data were Generated by Adding Random Deviations to the Earned and Actual Cost Profiles: (a) Cost Rate Profiles, (b) Cost Profiles, (c) Duration and Cost Estimates, and (d) Errors in the Estimates.

A useful summary of the accuracy of the estimates is that the error is high initially (the first 10% of the project) but quickly decreases and is well below 10% when the project is at 20% of the final actual duration. These results are consistent with both the theoretical estimate in equation 14 and many practical studies of real-world projects (Mamghaderi et al., 2021). We can also explain the observation that ES accuracy suffers towards the end of projects with S-shaped profiles; see the slight uptick in the grey line in Figure 5(d). S-shaped profiles flatten towards the end of the project and small deviations in the earned value significantly affect the intersection point of the projection back to the planned value curve, which is also flat at that point. Fortunately, this is not a major issue in practice as the true duration is known by

the time this effect arises. Real-world activities can have both random and systematic deviations. If the deviations are random, fitting the project data to a profile average the deviations, which should improve estimation accuracy, especially as more data become available as the project proceeds. There may also be systematic deviations, e.g., activities experiencing increasing delays. The impact of systematic deviations on estimation accuracy is a topic for future research. All of the above error analyses can also be applied to the earned cost method.

5. Conclusion

5.1. Theoretical Research Findings & Implications

As one can see from Figure 1, from any earned value data point one can draw four arrows that intersect the planned and actual cost curves. The two vertical arrows define traditional EVM quantities: cost variance and schedule variance. The two horizontal arrows define the earned schedule and the earned cost. Therefore, all intersections from any earned value data point have been defined; there are no more quantities to be defined. Also, there is no reason to prefer any one arrow over another: they are equally valid. The implication of this research finding, from a theoretical perspective, is that adding earned cost to EVM completed the theoretical development of EVM by establishing definitions for all fundamental quantities. The earned schedule and earned cost are defined by the fundamental relations in equations 3 and 9. This led to the derivation of cost and duration estimation formulas, which were proven to be universal: the estimation formulas are valid over the entire life of the project and for all projects in all industries. The implications of this research finding are that because the method was based on standard EVM definitions, and no approximations were required, the results are exact and generally applicable to all existing research. Therefore, while the practical utility of the EVM and ES methods has never been in doubt, this research finding considerably widens the general theoretical validity of EVM.

Equations 3 and 9 also established the basic theory that explains the behavior of the errors in the estimation accuracy. The key research finding is that when the planned and earned project data lie exactly on the planned and earned cost profiles, the functional relations in those equations guarantee that the estimates of the final duration and cost are exact. The valuable implication is that estimation errors are due to deviations between the project data and the cost profiles. Two types of errors were defined, and their size estimated. The implications of those errors were discussed, along with suggested practical approaches to reduce them.

The cost estimate derived from the EC method was proven to be equivalent to that of the traditional cost estimate at completion. An important property of the EC method is that the results are universal -- they apply to all projects. Therefore, because the CEAC was derived from the EC method, the traditional CEAC was also proven to be universal. To our knowledge, this is the first proof that the CEAC is universally applicable. We provided a simulation of project data, finding that the cost and duration results agreed with the theoretical predictions. Figure 5 also demonstrated that the accuracy of the earned cost, earned schedule, and estimate at completion methods was comparable. This agrees with several previous studies (Mamghaderi et al., 2021).

The earned schedule method traditionally uses cumulative data. However, it was shown, using a novel graphical method, that the calculation of the earned schedule can also be accomplished using planned and earned *rate* data. This provided a visual example that explained why the estimation formulas do not depend on the number of activities in each tine interval, i.e., that the estimation formulas apply to all projects. The visual method led to an additional research finding: that the earned schedule and critical path methods are formally identical. The implications of this are significant because the ES and CP methods have often been described as independent techniques for obtaining the final duration. That they are inherently related could significantly change the view of these techniques.

5.2. Practical Research Findings and Implications

These powerful theoretical research findings should have significant practical implications. Project managers can confidently use the duration and cost estimation formulas at any time on any project with the assurance that the resulting estimates were accurate and timely. The timely attribute means that the predictions are available early enough in the project's history for project managers to make appropriate managerial corrections. This was proposed by Teicholz (1993), who pointed out that project managers desire accurate forecasts in the early stages of the project. The many practical studies on the accuracy of cost and duration predictions are now legitimized by the proof that the estimation formulas apply to all projects.

If a project manager estimates a duration delay, the next step is to determine if it is worth any extra effort to analyze the results further. For example, on large projects with penalties for late delivery, extra effort may be justified to improve the accuracy of the predictions. In which case, the project manager could determine a suitable cost profile from a best fit to the planned data. Several authors have taken the extra step of fitting project data to a cost profile and concluded that using the resulting fit profile more accurately predicted the cost and duration (Evensmo & Karlsen, 2006). A reasonable fit to an appropriately selected cost profile should minimize the deviations between the project data and the cost profiles. The research finding that smaller deviations result in more accurate predictions implies that by generating good fits to cost profiles those authors reduced the deviations between the profiles and the data, thus improving duration and cost estimation accuracy.

5.3. A Note on Earned Duration

The difference between the earned duration (ED) and ES methods is that all ED activities, or tasks, have planned and earned values of one unit, whatever their planned value, earned value, or actual cost. For example, if a task is executed over a time, *t*, the earned duration contribution is 1/*t* in each time interval. Apart from the different method of counting the planned and earned value, the basis of the ED method is identical to that of the ES method: At time, *t*, one defines the planned duration as, $D_p(t)$, and the earned duration as, $D_e(t)$. We can then define the earned duration delay, $\delta_d(t)$, as the time interval from the current earned value to the horizontal intersection with the planned value, at $t - \delta_d(t)$:

$$D_e(t) = D_p[t - \delta_d(t)] \quad \text{with} \quad E_d(t) = t - \delta_d(t) \quad (16)$$

The earned duration, $E_d(t)$, is therefore, the time from the start of the project to that of the above intersection. By following the approach for ES and EC, one immediately arrives at:

$$T_f(t) = \frac{t\ T_p}{E_d(t)} \quad (17)$$

Using the ED method, it is presented that an estimation formula for the determination of the final project duration forecast (EDAC):

$$EDAC(t) = AD + \frac{BPD - E_d(t)}{PF} \quad (18)$$

where *AD* is the actual duration at the status date and *PF* is a performance factor. PF is chosen as the ED-based Duration Performance Index ($DPI = E_d(t)/t$), whereupon equation 18 gives,

$$T_f(t) = t + \frac{T_p - E_d(t)}{E_d(t)/t} = \frac{t\ T_p}{E_d(t)} \quad (19)$$

Thus, the ED method generates the same formula for the duration estimate as that obtained from the standard ES method (with the earned duration, $E_d(t)$, substituted for the earned schedule, $E_s(t)$). This follows because the definition of earned duration, equation 16, is identical to the definition of earned schedule, equation 3. The difference is that ED uses a different accounting method for the determination of the planned and earned value. In fact, all of the techniques of this paper should apply to the ED method. In particular, we conjecture that the ED method is universal and that, along with earned schedule and earned cost, is an integral part of a comprehensive Earned Value Management. The ED method has been shown to be slightly more accurate than the ES method in many cases (Mamghaderi et al., 2021). We can also conjecture that when using the ED method, the deviations between the data and the ED profiles are smaller than when using ES profiles, which results in more accurate duration estimates. It is a future research project to confirm these conjectures.

5.3. Avenues for Future Research

As per prior literature, it is concluded that all methods of schedule forecasting are good enough for practical work, but that the accuracy of ED was generally slightly superior. A prior study reached a similar conclusions while Mamghaderi et al. (2021) convincingly established that the earned duration (ED) method was often as accurate and, occasionally, more accurate, than the ES method. While these studies offer useful practical advice, they do not explain the relative accuracy characteristics of ED vs. ES. Since the ED method is founded on the same principles as those presented here for ES and EC, an interesting future research project could be to analyze the deviations for ED and ES and to assess the impact on the estimation accuracy.

5.4. Summary

We presented an algorithmic proof and a practical demonstration that the earned schedule-based and earned cost-based estimation formulas are universal. While the derivations are moderately complex, the estimation formulas are simple and the results theoretically powerful. The earned cost was integrated into EVM enhancing its theoretical development. Project managers can confidently use the universal duration and cost estimation formulas on any project.

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Appendix

A.1. Universality of the Duration Estimation Formulas

We define the three costs in terms of a general cost profile, *P*(*t*):

$$C_p(t) = B\,P\left(\frac{t}{T_p}\right), \quad C_e(t) = B\,P\left(\frac{t}{T_f}\right), \quad C_a(t) = C_f\,P\left(\frac{t}{T_f}\right) \quad (20)$$

Equation 20 makes explicit the assumption that the same functional form is used for the planned value, earned value, and actual cost. This formalizes the usually unstated assumption that EVM uses the same profile for all cost profiles, e.g., early presentations of EVM presented the same linear profiles for planned, earned, and cost. The planned cost, *B*, and the final actual cost, C_f, are taken outside of the cost profiles.

The important step is that the time-dependence of the profiles is written in dimensionless units: the planned value in terms of t/T_p and the earned value and actual cost in terms of t/T_f. Applying the profiles to the schedule delay, equation 3, gives:

$$B\,P\left(\frac{t-\delta_s(t)}{T_p}\right) = B\,P\left(\frac{t}{T_f}\right) \quad (21)$$

For any function, the left- and right-hand sides were equal for all times if the arguments of both functions are equal, which is the same as equation 3. Therefore, the same duration estimation formula emerges for all profiles, *P*(*t*). From Figure 1, one can observe that the earned schedule always exists because, at the end of the project, the total earned value equals the total planned value (PMI, 2011), so one can always draw a line from the earned to the planned curve. When the project is accelerated, $\delta_s(t)$ simply changes sign and equation 3 remains valid.

Rewriting the Cioffi profile in the form of equation 21 guarantees that the earned schedule and earned cost methods work for that profile. This is simply a redefinition of one of the Cioffi profile's parameters and doesn't invalidate any of the associated research.

A.2 Universality of the Cost Estimation Formulas

Using the same cost profiles as above and applying the definition of the cost delay,

$$C_f\,P\left(\frac{t-\delta_c(t)}{T_f}\right) = B\,P\left(\frac{t}{T_f}\right) \;\Rightarrow\; C_f = B\,\frac{P(t,T_f)}{P(E_c(t),T_f)} \quad (22)$$

where the earned cost, $E_c(t) = t - \delta_c(t)$, was substituted. This result is true for all cost profiles. We multiply the numerator and denominator by C_f, giving:

$$C_f = B\,\frac{C_f\,P(t,T_f)}{C_f P(E_c(t),T_f)} = B\,\frac{C_a(t)}{C_e(t)} = \frac{B}{CPI(t)} \quad (23)$$

where we have used the definition of the earned cost to substitute the earned value, $C_e(t)$, which is equal to the time delayed actual cost, $C_a(E_c(t))$. Equation 23 is the standard cost estimate at completion formula, *CEAC* (*t*). Because all equations are true for all times and all projects, the result is that the standard EVM formula for the *CEAC* (*t*) is proven to be valid for all projects.

In the special case of linear cost profiles:

$$C_f(t) = \frac{t\,B}{E_c(t)} \quad (24)$$

In Figure 1, the actual cost curve is above the earned curve indicating a cost overrun. In that case one can always draw a line from the earned data to the actual cost curve. When the actual cost curve is below the earned curve (a cost underrun), one can only draw a line up to the end of the actual cost curve. However, that is not a problem as that is the end of the project.

A.3. Deviation Errors

An estimate of how deviation errors affect the accuracy of the duration estimate can be made as follows. At time, *t*, suppose the deviation of the earned value is ΔC_e, see Figure 4. The error in the duration is then:

$$T_f + \Delta T_f = \frac{t\,T_p}{E_s(t) + \Delta E_s(t)} = \frac{t\,T_p}{E_s(t)}\left(1 - \frac{\Delta E_s}{E_s(t)}\right) \quad (25)$$

Assuming a small deviation $\Delta C_e \ll Ce(t)$), this simplifies to,

$$\frac{\Delta T_f}{T_f} \approx -\frac{\Delta E_s}{E_s(t)} \quad (26)$$

The minus sign indicates that the error in the duration is opposite in direction to that of the deviation. In Figure 4, the slope of the small triangle, with labels ΔCe and ΔE_s, is also the slope of the C_p (t) curve, which gives,

$$\frac{\Delta T_f}{T_f} \approx -\frac{\Delta C_e}{\Delta E_s} \quad (27)$$

Combining the above equations, and using the linear form of C_e (t), gives,

$$\frac{\Delta T_f}{T_f} \approx -\frac{\Delta C_e}{C_e(t)} \quad (28)$$