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ON UNCERTAINTY AND SENSITIVITY ANALYSES IN PROJECT DURATION BASED ON DEPENDENCY INFORMATION

ABSTRACT

The dependency structure matrix (DSM) shows the interdependency between activities, and it has been shown to be useful in the estimation of complex projects' durations. The estimate of project durations is based on activity durations, their interrelationships, and the permitted level of overlapping, all of which are represented by DSMs. However, these variables have individual uncertainties that generate overall uncertainty in the project duration. The objective of this work is to show that uncertainty analysis and sensitivity analysis are essential parts of analyzing the uncertainty in project scheduling. Specifically, this work shows how to perform sensitivity analysis in project scheduling using DSM, how to reduce the number of input variables with uncertainty for sensitivity analysis, and how to identify input variables whose control of uncertainty reduces the uncertainty of the project duration. An example is used to explain the methodology, and a case study is used to show the usefulness of sensitivity analysis and uncertainty analysis.

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1. Introduction

Uncertainty analysis refers to the determination of the uncertainty in output variables that derives from uncertainty in input variables, whereas sensitivity analysis refers to the determination of the contribution of the individual uncertainties of input variables to the uncertainty in output variables. Both uncertainty analysis and sensitivity analyses are essential parts of analysis for complex systems (Helton et al.,2006). Uncertainty analysis is usually conducted to identify risk (usually in project duration) due to unexpected events (currently considered negative events). Sensitivity analysis is not common in project scheduling. However, sensitivity analysis can be useful in the identification of unexpected events (both negative and positive) that produce risks and opportunities in project scheduling, which will be shown later.

In general, there are two types of uncertainty: epistemic (or systematic/reducible) and stochastic uncertainty (or aleatory/irreducible). Epistemic uncertainty is related to the absence of knowledge or incomplete knowledge about the appropriate value to use for a quantity that is assumed to have a fixed value in the context of a particular analysis. For example, the duration of an activity can be difficult to define if there is not a model that can predict its exact value. Conversely, stochastic uncertainty arises from the random behavior of the system under study. Activities that depend on weather conditions and human performance generally present this type of uncertainty. The uncertainty under consideration in this work that arises from the duration of the activities, their interrelationships, and the permitted level of overlap can be epistemic or stochastic.

Project scheduling, i.e., determining the sequence of activities, is important to the development of any project because appropriate sequencing reduces the amount of time necessary for completion. Project scheduling can be a difficult task because the order of activities is influenced by the information flow among them and because the activities can present overlap. The DSM can be used to model dependency (information flow) and interaction (overlap) between activities, which is why the DSM has been used to manage complex projects. In representing a project using DSM, different activities are broken down or are put together as needed to simplify the complexity of the project. For example, by breaking down the project into smaller activities and by identifying the relationship between them, it is possible to assess the effects of these smaller activities on the project and to assign resources to these activities (Browning, 2001). Several authors have recognized the importance of project scheduling to optimize the efficiency of the project (Tienda and Romano, 2011), an issue that also depends on the project analysis. In this sense, the DSM is a good tool to analyze the dependencies and interdependencies of a project, and therefore, recent efforts to reconcile project scheduling and DSM have sought to produce a tool that serves two purposes: analysis and project scheduling (Srour et al., 2013; Maheswari and Varshese, 2005). These studies have demonstrated that DSM is also a powerful tool in planning the sequence of activities.

However, activities in a project are subject to many unknown factors (Herroelen and Leus, 2005; Perminova et al., 2007) that can lead to changes in scheduling (Lamas and Demeulemeester, 2016). The factors that produce uncertainties have different origins, such as the unavailability of resources, the availability of materials before or behind schedule, and issues related to labor, with or without the desired qualifications, but all make activities take more or less time than was originally estimated (Fu et al., 2015). These uncertainties can cause the schedule to be delayed, increase stock, or require major work, all of which lead to higher costs than those originally planned. The information used by the DSM, including activity duration, the time required for communication, and activity overlap, can have uncertain values. The effect of these uncertainties on the project duration must be studied, and the identification of significant and insignificant input variables on the project duration uncertainty needs to be identified. Gálvez et al. (2015) applied Morris and Sobol´ methods to identify the key activities that affect the uncertainty in the duration of the project.

The objective of this work is to show that uncertainty analysis and sensitivity analysis are essential parts of analyzing the uncertainty in project scheduling. Therefore, some methods are introduced, but a complete review or survey of methods is outside of the objective of this manuscript. Overviews of uncertainty and sensitivity analysis are available in several reviews (Ionescu-Bujor and Cacuci, 2004; Cacuci and Ionescu-Bujor, 2004; Frey and Patil, 2002; Iooss and Lemaître, 2015) and books (Saltelli et al., 2008; Ronen, 1988; Saltelli et al., 2009). This paper shows that global sensitivity analysis (GSA) can be used to identify significant and insignificant input variables on project duration using the dependency structure matrix (DSM). This information can be used to identify input variables whose control is significant in reducing uncertainty in the project duration and to identify the input variables whose uncertainty is insignificant in sensitivity analysis.

Throughout this manuscript, a "simple example" is used to illustrate our discussion of uncertainty and sensitivity analyses. The example consists of six activities, A through F, and the DSM representation of the example is given in **Figure 1**. The example is "simple" because it has few activities, but the example is complex because there are several interactions between the activities and because there is uncertainty in the input variables. The DSM is a matrix of the same number of rows and columns, where activities are represented in the rows and columns with the aim of showing dependence. (For more information, see the work of Maheswari and Varghese, 2005.) Values along the diagonal are the mean duration time of the activities (days); for example, the mean duration of activity A is 2 days in **Figure 1**. The marks in the off-diagonal cells indicate that these activities are information predecessors, with activity inputs in the rows and activity outputs in the columns. Activity B needs information from activity A, and activity B provides information to activities D and E.

In **Figure 1**, the traditional, conventional, or sequential method for delivering a construction project is shown. In this method, an activity starts once its predecessors are completed. Based on the mean duration time of the activities, the conventional project duration is estimated to be 14 days (**Figure 1**). Note that activity C has no effect on the project duration, and all other activities are in the sequence of execution without any time left over between activities. The conventional project duration is estimated with

$$(EF)_i = (ES)_i + A_{ii} \qquad 0<i\leq n \qquad (01)$$

$$(ES)_j = Max[(EF)_i] \qquad 0<i\leq n, 0<j\leq n \qquad (02)$$

$$\text{Conventional project duration} = Max[(EF)_j] \qquad 0<j\leq n \qquad (03)$$

Equation 1 indicates that the early finish (EF) of activity i is determined based on the early start (ES) and on the duration (given by the diagonal values of DSM, A_{ii}). The $(ES)_j$ is the maximum of all $(EF)_i$, where i is the predecessors of j, and j is the current activity. Finally, the conventional project duration is the maximum value of EF of the n activities that comprise the project.

	A	B	C	D	E	F
A	2					
B	X	4				
C	X		3.5			
D		X	X	5		
E		X			5	
F				X	X	3

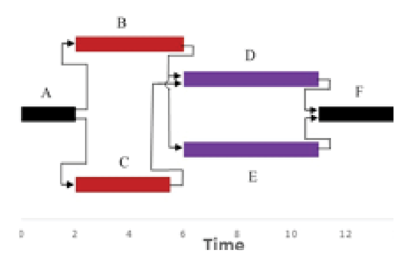
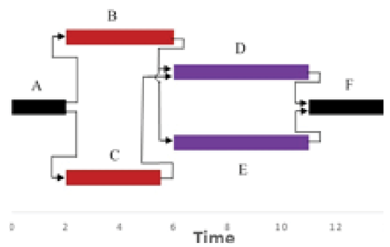


FIGURE 01. *DSM showing the mean values of the duration of activities (matrix A_{ij}) and conventional scheduling*

Figure 2 shows the fast-track or overlap method for delivering a project. In this method, some overlap occurs between pairs of activities. **Figure 2** uses the same example from **Figure 1** but allows for overlap between activities. The overlap is represented in DSM in the form of ratios called time factors (Maheswari and Varghese, 2005). Two time factors are used: the time factor of receiving the information for the successor activity (represented by matrix B_{ij}, given by the off-diagonal cell in **Figure 2b**) and the time factor of sending the information from the predecessor activity (represented by matrix C_{ij}, given by the off-diagonal cell in **Figure 2c**). In other words, an element of matrix *B* (for example B_{CA}) indicates the fraction of the duration of the predecessor activity (activity *A* with a fraction of 0.95 in **Figure 2**) at which that activity can send information to the successor activity (activity *C*). In the same way, an element of matrix *C* (for example C_{CA}) indicates the fraction of the duration of the successor activity (activity *C* with a fraction of 0.05 in **figure 2**) at which that activity needs to receive information from the predecessor activity (activity *A*). Based on the mean values of the activity durations and the mean values of the factor time, the natural overlap project duration is estimated to be 12.4 days (**Figure 2a**).



a) Overlap scheduling

	A	B	C	D	E	F
A	2					
B	0.87	4				
C	0.95		3.5			
D		0.95	0.95	5		
E		0.95			5	
F				0.95	0.95	3

b) Mean time factor of processor activities (B_{ij})

	A	B	C	D	E	F
A	2					
B	0.13	4				
C	0.05		3.5			
D		0.05	0.05	5		
E		0.05			5	
F				0.05	0.05	3

c) Mean time factor of receiving informatio[n] (C_{ij})

FIGURE 02. *DSM showing the mean values of the duration of activities and time factors of transfer of information between activities with overlap scheduling*

The natural overlap project duration is estimated with

$$(ES)_j = Max\,[(ES)_i + B_{ji}\,B_{ii} - C_{ji}\,C_{jj}] \qquad 0<i\leq n, 0<j\leq n \qquad \textbf{(04)}$$

$$(ES)_j = (ES)_i +_{ii} \qquad 0<i\leq n \qquad \textbf{(05)}$$

$$natural\ overlap\ project\ duration = Max[(EF)_j] \qquad 0<j\leq n \qquad \textbf{(06)}$$

where the meaning of the variables is the same as in equations 1 to 3, and the diagonal values of *C* DSM (B_{ii} and C_{ii}) are the duration of activity *i*.

2. Uncertainty analysis

The goal of this section is to show how to apply uncertainty analysis in project scheduling using DSM. The uncertainty analysis must answer the question: What is the uncertainty in the project duration (and other output variables, such as early finish) given the uncertainty in the activity durations and the time-overlap factors? Uncertainty analysis can be divided into three steps: **a)** First, it is necessary to represent the uncertainty based on the characteristics and behavior of the uncertainty. **b)** Then, the uncertainty in the output variables is determined based on the system model and the representation of the uncertainty of the input variables. **c)** Finally, it is necessary to represent the uncertainty of the output variables for analysis. These steps are briefly reviewed in the example above.

2.1 Representation of uncertainty

The uncertainty of the input variables can be characterized using fuzzy theory, grey theory (interval analysis), and probabilistic theory, among others. Grey and fuzzy theories are best suited to represent epistemic uncertainty, whereas the theory of probability is more suitable for the stochastic uncertainty. Grey and fuzzy theories differ in the meaning and extension of uncertainties. In fuzzy theory, meaning is clearly defined (the duration of this task is long), but its extension is represented by membership functions that are diffuse. By contrast, the grey theory extension is clearly defined (the activity lasts between 2 and 3 days), but its meaning is not explicitly stated. In probability theory, uncertainty is represented by probability distribution functions. In this work, only the probabilistic representation of uncertainty is considered. However, several works have addressed the representation of uncertainty based on the work of Maheswari and Varghese, (2005) on project scheduling using DSM. Gálvez et al. (2012) studied the effect of the uncertainty of activity programming using DSM and grey theory or interval arithmetic. Shi and Blomquist (2012) extended the DSM method proposed by Maheswari and Varghese using fuzzy numbers. The drawback of these methods is the need to characterize all input variables, which are typically defined through an expert review process, and overestimation of the uncertainty of the output variables has been demonstrated for interval arithmetic (Gálvez et al., 2015) and is shown here for fuzzy numbers.

The definition of the distribution functions that represent the uncertainty in the duration of activities and the time-overlap factors can be one of the most significant parts of uncertainty analysis because these distributions can determine the uncertainty of the project duration. The distribution functions that represent the stochastic uncertainty in input variables can be defined based on the data available from previous projects. To use these data to identify the distribution function, it is necessary to know whether the conditions under which these data were obtained are the same conditions under which the project will be analyzed.

The distribution functions to represent the epistemic uncertainty must be defined through an expert review process, and their development can constitute a major analysis cost. The process of extracting expert knowledge about some unknown quantity or quantities and formulating that information as a probability distribution is known as elicitation (O´Haga et al., 2006; Meyer, 2006,). The range of elicitation can vary broadly based on the purpose and size of the analysis and the resources available to perform the analysis. An alternative is to perform an initial uncertainty analysis based on a crude definition of the distribution functions for the activity duration time and the time overlap factors to understand the behavior of the project duration uncertainty. Then, resources can be concentrated where needed. Sensitivity analysis can also be used to reduce the number of input variables before the characterization, as will be shown later.

Defining these distributions by specifying their parameters (mean, standard deviation) is not advisable; rather, selected cumulative distribution functions should be specified based on the experts. Here, the distributions are specified using their parameters as the first step in the analysis. This work uses three types of distribution functions for the input variables to study their effect on the uncertainty in the project duration. The distribution functions studied have a continuous uniform distribution, a normal distribution, and a log-normal distribution. The continuous uniform distribution is here abbreviated as U(a,b), where a and b are its minimum and maximum values, respectively. The normal distribution is abbreviated as N(μ,σ^2), where μ is the mean, and σ^2 is the variance. The log-normal distribution is abbreviated as log-N(μ, σ^2), where μ and σ^2 are the mean and the variance of the variable's natural logarithm, respectively.

For a uniform distribution, each duration activity has an uncertainty of ±0.5 days, which indicates, for example, that activities *A* and *D* have durations of U(1.5,2.5) and U(4.5,5.5), respectively. Similarly, each time factor has an uncertainty of ±0.05, so the off-diagonal values of B_{ij} are U(0.9,1.0), and the off-diagonal values of C_{ij} are U(0.0,0.1), but B_{BA} is U(0.74,1.0), and C_{BA} is U(0.0,0.26). For a normal distribution, each duration activity has a variance of 0.0841 (a standard deviation of 0.29) and a mean given in **Figure 1**; for example, activities *A* and *D* have durations of N(2,0.29^2) and N(5,0.29^2), respectively. For a log-normal distribution, each duration activity has a variance of 0.09 (a standard deviation of 0.3) and a mean given by the logarithm of the values given in **Figure 1**; for example, activities *A* and *D* have durations of log-N(log(2),0.3^2) and log-N(log(5),0.3^2), respectively. Likewise, each time factor has a variance of 0.04 (a standard deviation of 0.2) and a mean given by the logarithm of the values given in **Figure 2**. The off-diagonal values of B_{ij} are log-N(log(0.95),0.2^2), and the off-diagonal values of C_{ij} are log-N(log(0.05),0.2^2), whereas B_{BA} is log-N(log(0.87,0.2^2), and C_{BA} is log-N(log(0.13),0.2^2).

2.2 Determination of output variables' uncertainty

In fuzzy and grey theories, the determination of uncertainty in project duration and other output variables is performed using fuzzy mathematics and interval mathematics. In other words, the mathematical operators in equations 1-6 are replaced by fuzzy or grey transformations. The works of Gálvez et al. (2012) and Shi and Blomquist (2012) show these transformations for grey and fuzzy theories, respectively. In probability theory, the determination of the output variable uncertainty is performed in two steps, the generation of the sample and the evaluation of the output variables for each sample element.

2.2.1 Sample generation

Random sampling is the simplest form of sampling because new sample points are generated without taking into account the previously generated sample points. However, a great number of samples are typically required in random sampling to achieve good accuracy, which is why other sampling strategies have been developed. The more common sampling strategies, different from random sampling, are importance sampling, orthogonal sampling, and Latin hypercube sampling. Latin hypercube sampling is a widely used method to generate controlled random samples (Mckay et al., 1979) because its effective stratification properties permit the extraction of a large amount of uncertainty information with a relatively small sample size. Latin hypercube sampling should be considered a good option when the model has a high computational cost. Orthogonal sampling adds to the stratification properties of Latin hypercube with the requirement that the entire sample space must be sampled evenly. Orthogonal sampling is more efficient but more difficult to implement. Importance sampling is more effective for large sample sizes to cover the low probability and/or high consequence subsets of values for input variables and usually is not a good option for sensitivity analysis. Because the models for project duration are simple, random sampling can be an adequate sampling strategy, but if a large number of activities are involved, then Latin hypercube can be a more efficient alternative. Latin hypercube sampling is also available in several commercial software programs. Here, random sampling was used with 3,000 model calls for the simple example and with 100,000 model calls for the case study (a medium-sized problem).

2.2.2 Evaluation of the output variables for each sample element

Usually, the evaluation of the project duration and other output variables for each element of the sample is the step that demands the most computational costs. This step aims to determine the project duration and the *ES* and *EF* of each activity for each value of activity duration and overlapping factors of the sample. In project scheduling, this process is simple because the models are simple. Equations 1 through 3 are for a conventional project duration, and equations 4 through 6 are for a natural overlap project duration. In these models, the maximization functions produce discontinuities in the model, and these discontinuities do not allow for the use of local sensitivity analysis because the equations are not differentiable. Furthermore, the models are monotonic.

2.3 Presentation of output variable uncertainty

The presentation of uncertainty analysis results involves little more than displaying the results associated with the calculated mapping of the activity duration time and the time overlap factors versus the project duration. For fuzzy and grey theories the output variables are presented by fuzzy numbers and intervals, respectively. For probability theory, the options for presentation include means and standard deviations, density functions, cumulative distribution functions, box plots and statistical tests (Tufte, 2001).

Table 1 gives the median, mean, minimum, maximum, and first and third quartiles. These values give us a first view of the uncertainty in the project duration. The median and mean are central values, and for normal distribution, the mean and median are actually equivalent. Based on the values in **Table 1**, the normal and uniform distributions have similar median and mean values, but for the log-normal distribution, the mean is greater than the median. In addition, the uniform and normal distributions have similar behavior.

Figure 3 shows the histogram and density function for the conventional and natural overlap project durations. When the uncertainty in the activity durations and time-overlap factors are represented by uniform and normal distribution functions, the uncertainty in project durations follows a normal distribution function. This visual behavior is confirmed by the Kolmogorov-Smirnov test of normality (Stephens, 1974). When the uncertainty in the activity durations and the time overlap factors are represented by a log-normal distribution function, the uncertainty in the project durations does not follow a normal distribution function. However, if the standard deviation of the log-normal distribution of the input variables is reduced to half its value, then the uncertainty in the project durations follows a normal distribution with a mean and a median of 13.07 and 13.05, respectively.

The normal distribution is symmetric, which means that the probability that the project duration is one day more than the mean is the same as the probability that the duration is one day less than the mean. The log-normal distribution (**Figure 3c**) shows that the project is more likely to have a longer duration than the mean than a shorter one.

The cumulative distribution function of the project duration describes the probability that the project duration will have a value less than or equal to a specific value of project duration. **Figure 4** shows the cumulative functions for the conventional project duration. Normal distribution

	Conventional project duration			Natural overlap project duration		
	Uniform	Normal	Log-Normal	Uniform	Normal	Log-Normal
Minimum	12.51	12.57	9.64	11.03	9.5	8.37
1st Quartile	13.78	13.8	14.21	12.35	12.31	13.05
Median	14.16	14.19	15.58	12.74	12.92	14.48
Mean	14.17	14.19	15.80	12.75	12.95	14.79
3rd Quartile	14.55	14.57	17.24	13.14	13.56	16.29
Maximum	15.76	15.77	25.43	14.67	16.44	27.46

TABLE 01. *Basic statistics for project duration uncertainty*

functions are also included, but there is no difference with the cumulative functions, except for the log-normal distribution. The probability that the project duration has a value less than or equal to 16 days is 1, 1, and 0.45 for the uniform, normal and log-normal distributions, respectively.

A box plot or box and whisker is based on quartiles, and in it, a set of data is displayed. The box shows the first, second (median), and third quartiles, whereas whiskers represent the maximum and minimum values. Atypical values are shown with points. Specifically, the ends of the whiskers represent the lowest datum still within 1.5 of the interquartile range (IQR) of the lower quartile and the highest datum still within 1.5 IQR of the upper quartile. The IQR is a measure of statistical dispersion, equal to the 1st quartile subtracted from the 3rd quartile. Box plots may seem more basic than histograms, but they do have some advantages. They take up less space and are therefore particularly useful for displaying and comparing the uncertainty of a number of variables. For example, **Figure 5** shows the early start and early finish of all activities for the conventional project duration. Symmetry without an extreme observation in the box plots is typical of uniform distribution, whereas symmetry with extreme observation is typical of a normal distribution. Moreover, asymmetry in the box plots is typical of a log-normal distribution.

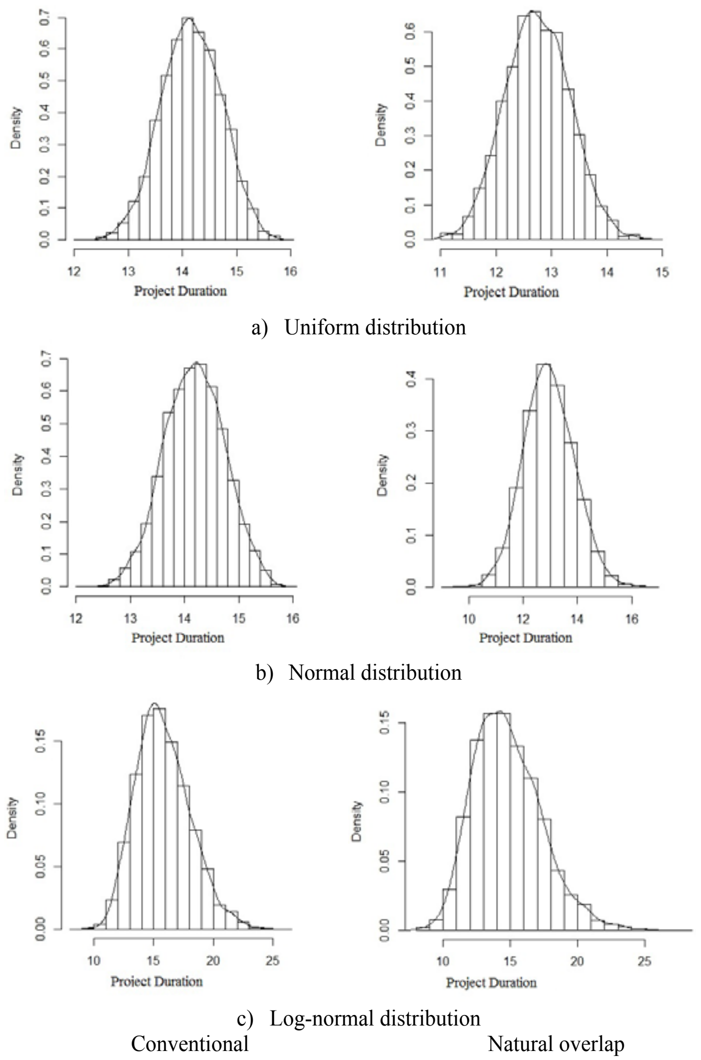


FIGURE 03. *Histogram and density functions for conventional and natural overlap project durations*

In **Figure 5a**, the early starts of activities *B* and *C* follow an uniform distributions, and the early finishes of activities *D* and *F* follow normal distributions. In **Figure 5b**, log-normal distributions are observed.

The box plots of *B* and *C* activities for the uniform distribution show that the probability that *C* ends after *B* is low (see the first and third quartiles of early finish, **Figure 5a**). Then, activity *B* controls the early start of activities *D* and *E*. A different behavior is observed for the log-normal distribution (**Figure 5b**), where there is a higher probability that *C* ends after *B*; therefore, activities *D* and *E* have different early starts.

3. Sensitivity analysis

The objective of this section is to show the use of the Sobol method, a global sensitivity method, for identifying significant and insignificant input variables on the project duration using the DSM. This information is used to identify input variables whose control is significant for reducing the uncertainty in project duration and to identify the input variables whose uncertainty is insignificant in sensitivity analysis.

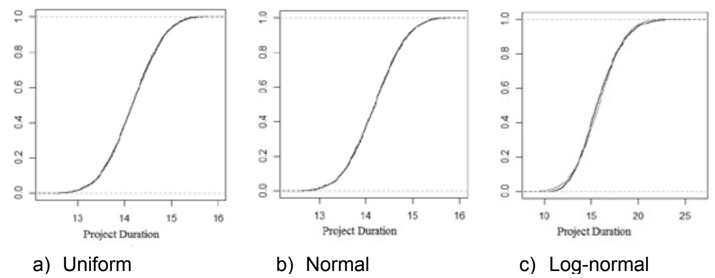


FIGURE 04. *Cumulative functions for the conventional project duration*

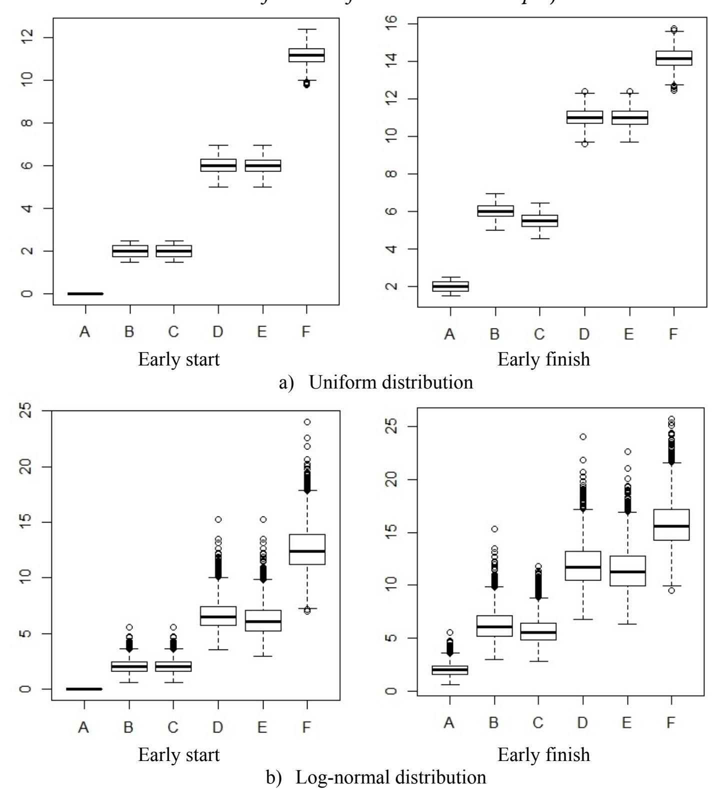


FIGURE 05. *Box plot for early start and early finish in conventional project duration*

3.1 Sobol method.

Sensitivity analysis refers to the determination of the contribution of individual uncertainty inputs to the uncertainty in output results (Helton et al., 2006). Saltelli et al. (2008) defined GSA as "the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input." One of the major applications of GSA is in the identification of significant or influential (and insignificant/non-influential) variables to use this information to reduce the number of variables with uncertainty for further studies. In addition, GSA can be used to understand model behavior, for example, by identifying interactions between variables and linear (or nonlinear) behavior between output and input variables. GSA, as opposed to local sensitivity analysis, corresponds to the evaluation of an output model when all input variables are simultaneously evaluated and considering the total uncertainty range. Although these two differences provide advantages to GSA in comparison with local sensitivity analysis, GSA requires more computational costs because it requires a large sample and more data manipulation of the sample (Sepúlveda et al., 2014). GSA methods can be classified into three groups (Confalonieri et al., 2010): regression methods, screening methods, and variance-based methods.

Variance-based methods decompose the variance of the model output into terms of partial variances, which denote the contribution of the inputs to the overall uncertainty in the model output. These partial variances are estimated using multi-dimensional integrals, which is computationally expensive. To reduce the computation cost, Homma and Saltelli (1996) developed the concept of a total sensitivity index. In the total sensitivity index, the overall effect of a given input is included by considering all possible interactions of the respective input with all other inputs.

There are several methods based on the variance that can be used to analyze project scheduling using DSM. Here, the Sobol' (1993) method and the improvements made by Jansen (1999) and Saltelli et al. (2010) will be used. Specifically, both GSA methods will be used to study how the uncertainty in project duration can be assigned to the activity duration and overlapping factor uncertainties.

Let us consider that the model has the form $Y=f(x_1,x_2,\ldots x_n)$, where Y is a scalar, and x_i is a model variable; then, the Sobol method is based on the partitioning of the total variance of the model output $V(Y)$ using the following equation (Confalonieri et al., 2010; Sepúlveda et al., 2014):

$$V(Y)=\Sigma^n_{i=1} D_i + \Sigma^n_{i\leq j\leq n} D_{ij} + \ldots + \Sigma^n_{i\leq\ldots n} D_{i\ldots n} \quad (07)$$

where D_i, D_{ij}, D_{ijk}, $D_{1\ldots n}$ denote the first-order effect, the second-order effect, the third-order effect, and the interactions among n variables, respectively, where $(D_i=V[E(Y|x_i)])$, $(D_{ij}=V[E(Y|x_i,x_j)]-D_i-D_j)$, $(D_{ijk}=V[E(Y|x_i,x_j,x_k)]-D_{ij}-D_{jk}-D_{ki}-D_i-D_j-D_k)$. The variance of the conditional expectation $(V[E(Y|x_i)])$ is sometimes called the main effect, and it is used as an indicator of the significance of x_i. In the Sobol method, the first-order effect sensitivity index corresponding to a single variable (x_i), S_i, is given by

$$S_i=\frac{V[E(Y|x_i)]}{V(Y)}=\frac{D_i}{V(Y)} \quad (08)$$

The second-order effect sensitivity index corresponding to two variables (x_i and x_j), S_{ij}, is given by

$$S_{ij}=\frac{D_{ij}}{V(Y)} \quad (09)$$

and so on until order *n*.

Based on equation 7 and the definition of the Sobol sensitivity indices (equations 8 and 9), the following relation can be obtained:

$$1=\Sigma^n_{i=1} S_i + \Sigma^n_{i\leq j\leq n} S_{ij} + \ldots + \Sigma^n_{i\leq\ldots n} S_{i\ldots n} \quad (10)$$

The interpretation of the sensitivity indices is straightforward: As the sensitivity index increases in size, the corresponding input variable or set of input variables becomes more influential. Because the indices are all positive, the maximum value of a sensitivity index is 1. The number of indices can become large (the total number is 2^n-1), and therefore, its interpretation can become unmanageable. Homma and Saltelli (1996) introduced the total sensitivity indices, which assess the sensitivity of the variance of the output variable with respect to the standalone and every interaction of the considered input variable, that is

$$S^T_i=1-\frac{D_{-i}}{V(Y)} \quad (11)$$

where D_{-i} represents the variance explained by all input variables except x_i $(D_i=V[E(Y|x_{-i}])$.

The first-order sensitivity index measures the main effect of each input variable's uncertainty to the output variance uncertainty. When the first-order sensitivity index (S_i) of the i input variable has a high or moderate value, the uncertainty in x_i affects the uncertainty in the output model, *Y*, and therefore, x_i is significant. Inversely, when the value of first-order sensitivity index (S_i) is zero or very small, x_i is insignificant (Sepúlveda et al., 2014). The first-order sensitivity index does not take into account the interaction among variables; therefore, it does not say anything about the input interactions or high-order sensitivity indices, such as $S_{i,j}$ or $S_{i,j,k}$. The total sensitivity index (S^T_i) is important when the objective is to reduce uncertainty in the output model (Adeyinka, 2007). If the total sensitivity index (S^T_i) is small, then apart from being insignificant, x_i does not interact with other variables, and high-order effects of x_i are negligible. Therefore, the uncertainty in x_i has no effect on the uncertainty in *Y* if S_i and S^T_i are small. Then, in successive analysis, x_i can be fixed to its nominal value, and further research, measurement, analysis and data gathering can be directed toward other variables (Sepúlveda et al., 2014). The interactions between x_i and other variables can be calculated by the arithmetic difference between S^T_i and S_i.

3.2 Application

Two methods are used here for the computation of Sobol indices: the Sobol standard estimator, which allows for estimating the indices of the variance decomposition up to a given order at a total cost of $(P + 1) * s$ model evaluation, where *P* is the number of indices to estimate and *s* is the sample size, and the Sobol-Jansen method for both first-order and total indices simultaneously at a total cost of $(n + 2) * s$ model evaluations, where n is the number of input variables. Software R (R Core Team, 2013) and package sensitivity (Pujol et al.,2014), which is a free software environment for statistical computing and graphics, were used.

The Sobol-Jansen method was applied to the example for conventional project duration (equations 1 through 3) with six random inputs (size of 50,000) with Monte Carlo sampling and a cost of 400,000 model calls. **Figure 6a** plots the first-order and total Sobol indices for a uniform distribution. It is easy to visualize that *A, F, B, D,* and *E* activities are influential in that order (with large values of both first-order and total Sobol indices), whereas *C* has no effect. In addition, *D* and *E* have interaction. (The total and first-order indices have different values.) The interactions in the other activities are small. Similar results were obtained for the normal distribution functions. **Figure 6b** plots the results for a log-normal distribution. Activities *D, B, E,* and *F* are influential in that order (with large values of both first-order and total Sobol indices), whereas *A* and *C* have little effect. In addition, *B, C, D,* and *E* have interactions. (The total and first-order indices have different values.) The interactions in *A* and *F* activities are small.

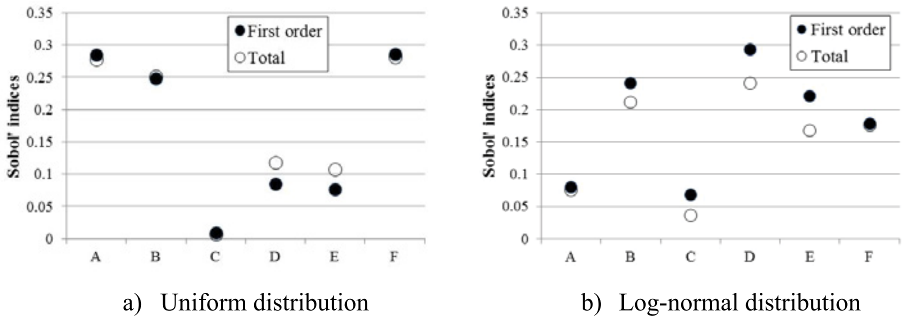


FIGURE 06. *Estimation of Sobol-Jansen indices for the example without overlap*

FIGURE 07. *Sobol standard estimator indices for conventional project duration*

As a comparison, the Sobol standard estimator method was also applied to the example for conventional project duration (equations 1 through 3) with six random inputs with Monte Carlo sampling and a cost of 1,100,000 model calls, which is 2.75 times more expensive than the Sobol-Jansen method. This cost is higher because the Sobol standard estimator has to determine 21 indices (six first-order and 15 second-order indices), whereas the Sobol-Jansen method has to determine only twelve indices (six first-order and total indices). **Figure 7** plots the first- and second-order indices for uniform and log-normal distributions. The first-order indices are similar to the ones obtained using the Sobol-Jansen method. The second-order indices show that the only important interaction is between the duration of activities *D* and *E* for the uniform and log-normal distributions. In addition, the log-normal distribution also shows the interaction between *B* and *C* activities, which is consistent with the Sobol-Jansen results.

The difference between the uniform (and normal) and log-normal distribution is due to the difference in the magnitude of the uncertainty in the time duration of each activity. This can be seen in **Figure 5** in the comparison of the box plots of the early start and early finish of the uniform and log-normal distributions. However, if the standard deviation of the log-normal distribution in the input variables is reduced to half its value, then the Sobol and Sobol-Jansen indices for the log-normal distribution follow a behavior similar to that of the uniform and standard distributions. A similar behavior was also observed in the project duration uncertainty analysis in the previous section.

The Sobol-Jansen method was applied to the example with overlap (equations 4 through 6) with 20 random inputs with Monte Carlo sampling and a cost of 1,100,000 model calls. **Figure 8a** plots the results for the uniform distribution. The *A, B, D*, and *F* activity durations and the C_{BA} time factor are very influential (with large values of the Sobol indices), and *C* and *E* activity durations and the B_{BA} and B_{FD} time factors are influential. Several time factors have no effect (with values of the Sobol indices close to zero). In addition, *B, C, D, E* and C_{BA} have interactions. (The total and first-order indices have different values.) When normal distribution functions are used to represent the uncertainties in the input variables, similar behavior is observed. **Figure 8b** plots the results for the log-normal distribution. Uncertainty in the durations of activities *D, B, E*, and *F* and the B_{-FD} and B_{FE} time factors are very influential (with large values of the Sobol indices), and the *A* and *C* activity durations and the B_{DB}, B_{CD}, and B_{EB} time factors are influential. Several time factors have no effect (with values of the Sobol indices close to zero). In addition, *B, C, D*, and *E* and B_{FD}, B_{FE}, B_{DB}, B_{CD}, and B_{EB} have interactions (the total and first-order indices have different values). The differences between the uniform and the log-normal distributions are due to the magnitude of the uncertainties used in each case.

The Sobol standard estimator method was not applied to natural overlap project duration because this case has 20 input variables, with a need to evaluate 2^{20}-1 indices, which is unmanageable.

3.3 Discussion

For the example of conventional project duration (without overlap), all activities have the same level of uncertainty in duration; however, the effect of these uncertainties on the uncertainty of the project duration is different.

For a uniform distribution, normal distribution, and log-normal distribution with the standard deviation reduced to half of its value, the following behavior is observed: The uncertainty in the time duration of activities *A* and *F* is the most relevant to the uncertainty in the project duration (with the largest values of Sobol indices) because these activities are sequential without interaction and will always influence project implementation. The uncertainty in the time duration of activities *D* and *E* also affects the uncertainty in the project duration. However, activity *D* affects only whether the duration of activity D is greater than the duration of activity *E*, and vice versa. For that reason, these activities have interaction (different values in the first-order and total Sobol indices).

These results indicate that efforts to reduce uncertainty in the project duration should focus on reducing the uncertainty in the duration of activities *A, F* and *B*. Reducing uncertainty in the duration of activities *D* and *E* has a smaller effect on the uncertainty in the project duration. Reducing uncertainty in the duration of activity *C* will have minimal effect. If resources are limited, the resources must be allocated to estimate the uncertainty of activities *A, F* and *B*.

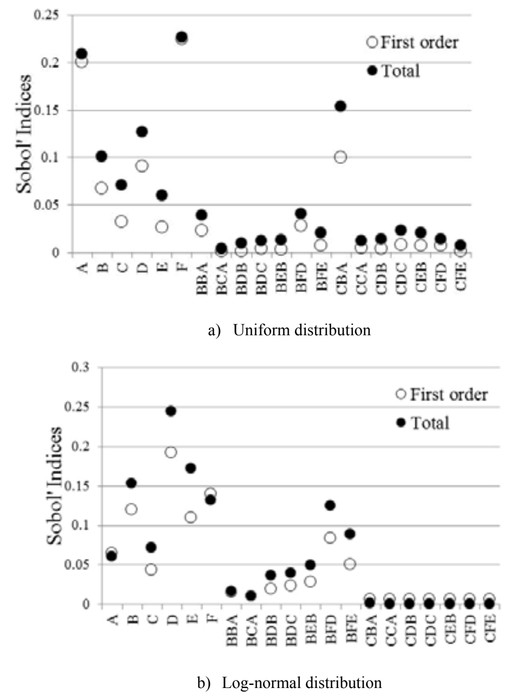


FIGURE 08. *Estimation of Sobol-Jansen indices for natural overlap project duration*

Table 2 shows the results of Monte Carlo simulations for various scenarios with 1,000 calls to the model. The second column shows the results of the project duration when considering uncertainty in all activities. Columns two, three and four show the results of the project duration when the duration of activities *A* and *F, D* and *E*, and *C* is fixed (at their average values), respectively.

Despite the uncertainty in the duration of each activity having a uniform or normal distribution, the project duration is normally distributed. Note that the average value of the project duration is larger than the value calculated with the mean values (14 days) because the interaction was not considered. In fact, if the activity durations with the largest interactions is fixed (*D* and *E*), the mean value is close to 14 days. If all activities are uncertain, then the uncertainty in the project duration is 3.4 days; if the uncertainty in activity *C* is removed, the uncertainty in the project duration is not significantly reduced at 3.3 days. However, if the uncertainty in activities *A* and *F* is eliminated, the uncertainty in the project duration is reduced to 1.8 days, compared to 2.7 days if the uncertainty is removed in activities *D* and *E*. This finding confirms that GSA (the Sobol method) can be used to identify the input variables and that if their uncertainties are reduced, then the uncertainty in the project duration is also reduced. The Sobol method can also be used to identify the input variables that do not affect the uncertainty in the project duration and can therefore be fixed at their mean values for uncertainty analysis.

The final decision on where to focus efforts on reducing the uncertainty depends on these results and on other aspects, such as the associated cost, the availability of resources and the feasibility of reducing uncertainty in the activity duration.

	Project duration			
	No activity fixed	A and F fixed	D and E fixed	C fixed
Minimum	12.41	13.15	12.74	12.53
1st Quartile	13.76	13.97	13.67	13.80
Median	14,16	14.19	14.03	14.20
Mean	14.16	14.19	14.02	14.19
3rd Quartile	14.52	14.47	14.39	14.59
Maximum	15.82	14.97	15.43	15.83

TABLE 02. *Uncertainty analysis in conventional project duration for various scenarios and uniform distribution for the input variables*

For the log-normal distribution, different results are observed, as shown in **Figure 6b**. The uncertainty in the time duration of activities *B, D, E* and *F* is the most relevant to the uncertainty in the conventional project duration. The difference in the behavior indicates that the characterization of the uncertainty in the input variables is a key component of uncertainty analysis. Furthermore, the magnitude (variation) of uncertainty is more influential than the form of uncertainty for the project duration.

Table 3 shows the results of Monte Carlo simulations for various scenarios with 1,000 calls to the model using a log-normal distribution for the input variables. The second column shows the results of the project duration when considering the uncertainty in all activities. Columns two, three and four show the results of the project duration when the duration of activities *B* and *D, A*, and *C*, and *B, D*, and *E*, respectively is fixed (in its average value). If all activities are uncertain, then the uncertainty in the project duration is 15.57 days; if the uncertainty in activities *A* and *C* is removed, the uncertainty in the project duration is not significantly reduced at 14.57 days. However, if the uncertainty in activities B, D, and E is eliminated, the uncertainty in the project duration is reduced to 10.05 days.

	Project duration			
	No activity fixed	B and D fixed	A and C fixed	B, D, E fixed
Minimum	10.19	11.76	10.43	11.28
1st Quartile	14.23	13.93	14.01	13.59
Median	15.07	14.89	15.30	14.36
Mean	15.82	15.12	15.53	14.53
3rd Quartile	17.20	16.07	16.80	15.28
Maximum	25.76	25.94	25.00	21.33

TABLE 03. *Uncertainty analysis for conventional project duration for various scenarios and log-normal distribution for the input variables*

In the example of a natural overlap project duration with an uniform distribution in the input variables, the time factors generally have less effect on the uncertainty in the project duration, with the exception of the time factor C_{BA} (see **Figure 8a**). This result is not surprising because C_{BA} is the time factor with the most uncertainty. However, the effect of the B_{BA} time factor is not as significant despite having high uncertainty because the effect of C_{BA} depends on the duration of activity *B*—whereas the effect of B_{BA} depends on the duration of activity *A* (see equation 4)—and because the duration of *B* is longer than the duration of activity *A*. If all activity durations and time factors have uncertainty, the uncertainty in the project duration is 4.1 days (based on Monte Carlo simulations); if the input variables that most affect the project duration uncertainty are fixed at their mean values (activities *A, F, B, D* and time factor C_{BA}), the project duration uncertainty is reduced to 2.0 days, showing a significant effect. If the duration of activities *C* and *E* is fixed, then the project duration uncertainty is 4.0 days; i.e., its effect is marginal. Conversely, if the duration of activities *A* and *F* is fixed, the uncertainty is 3.0 days; i.e., there is a significant effect. These simulations confirm that the Sobol method can identify which input variables in which to reduce uncertainty to reduce the uncertainty in the project duration. The Sobol method can also be used to identify the input variables that do not affect the uncertainty in the project duration and can therefore be fixed at their mean values for uncertainty analysis.

In the example of a natural overlap project duration with a log-normal distribution for the input variables, the time factors have less effect on the uncertainty in the project duration, with the exception of time factors BC_{FD} and BC_{FE} (see **Figure 8a**). If the input variables (activity duration and time factor) with the highest Sobol indices are fixed, the uncertainty in the project duration is reduced more significantly than if the other input variables are fixed. This is a result similar to that described for the previous cases.

Another application of the results obtained by applying GSA is uncertainty analysis. A possible analysis strategy is to perform an initial estimation of the distribution functions of the activity durations and overlap time factors and then use GSA to identify the most significant input variables. Now, resources can be concentrated on characterizing the uncertainty of these input variables, and a second uncertainty analysis can be performed with these improved uncertainty characterizations.

4. Case Study

In this section, a case study with 17 activities is analyzed. This example was presented by Shi and Blomquist (2012). Shi and Blomquist employed triangular fuzzy numbers to describe the uncertainty of the duration of activities and time factors. They used as a membership function a positive triangular fuzzy number F=(l,m,u):

$$u(x)=\begin{cases} \frac{x-l}{m-l}, l \le x \le m \\ \frac{u-x}{u-m}, m \le x \le u \\ 0, \text{otherwise} \end{cases} \quad (12)$$

For the application of GSA, these fuzzy numbers are used as triangular distribution functions. The dependent relationships of information between the activities are shown by the off-diagonal element of matrix B_{ij} in **Figure 9**. For example, the information predecessors of activity *E* are activities *A*, *B* and *C*. The duration of each activity is given in the diagonal element of matrix B_{ij}. To quantify the complexity of this case study, Shi and Blomquist (2016) calculated the project order strength to be 0.48 by describing the project as a network diagram with 18 nodes. **Figure 9** also gives the time factor of B_{ij}; three fuzzy numbers were used (0.8,0.9,0.9), (0.6,0.7,0.8), and (0.4,0.5,0.6) with transformations in linguistic variables given by Shi and Blomquist as "required information that can be released after almost all of the work has been finished", "required information that can be released after a large amount the work has been finished", and "middle status", respectively (Shi and Blomquist, 2016). The values of the time factor of C_{ij} are as follows: C_{DA}, C_{EB}, C_{EC}, C_{HE}, C_{IF}, C_{KG}, C_{KI}, C_{LI}, C_{PC}, C_{QP}, C_{RJ}, C_{RL}, C_{RN}, and C_{RQ} have a fuzzy number of (0.1,0.1,0.2); C_{EA}, C_{FC}, C_{IE}, C_{JG}, C_{KH}, C_{MA}, C_{ND}, C_{NM}, and C_{RK} have a fuzzy number of (0.2,0.3,0.4); and C_{GD} has a fuzzy number of (0.4,0.5,0.6). The transformation in linguistic variables of (0.1,0.1,0.2), (0.2,0.3,0.4), and (0.4,0.5,0.6) given by Shi and Blomquist is "a little work can be conducted before the information is released from its predecessor activity", "some work can be conducted before the information is released from its predecessor activity", and "middle status", respectively.

The uncertainty analysis was performed as described previously for the simple example. There are 65 input variables: 17 activity durations, 24 B_{ij} time factors, and 24 C_{ij} time factors. **Figure 10a** shows the histogram of the project duration determined by Monte Carlo simulation with 100,000 model calls. The minimum, maximum, and mean values are 16.81, 23.23 and 19.79, respectively, compared to 11.6, 27.4 and 20.4 using fuzzy numbers (Shi and Blomquist, 2012). This result is consistent with the observations for interval calculation (Gálvez et al., 2015); that is, interval calculation (and fuzzy numbers) overestimate the uncertainty in project duration.

Figures 11a and **11d** display the box plot of the early start and early finish for this example. The results are consistent with the values given by the fuzzy numbers (Shi and Blomquist, 2012), but overestimation occurs as more calculation steps are needed. This overestimation happens because the probability that all variables have the minimum (or maximum) values decreases as more variables are involved.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	P	Q	R
A	5 6 8																
B		3 4 6															
C			4 5 6														
D	0.8 0.9 0.9			2 3 4													
E	0.6 0.7 0.8	0.8 0.9 0.9	0.4 0.5 0.6		1 2 4												
F			0.6 0.7 0.8			2 3 4											
G				0.8 0.9 0.9			5 6 7										
H					0.6 0.7 0.8			3 4 6									
I					0.8 0.9 0.9	0.4 0.5 0.6			1 2 4								
J							0.6 0.7 0.8			6 7 8							
K							0.8 0.9 0.9	0.6 0.7 0.8	0.4 0.5 0.6		1 2 3						
L									0.8 0.9 0.9			2 4 5					
M	0.6 0.7 0.8												1 2 4				
N				0.8 0.9 0.9									0.6 0.7 0.8	2 3 4			
P			0.8 0.9 0.9												3 4 5		
Q															0.8 0.9 0.9	2 3 5	
R										0.8 0.9 0.9	0.6 0.7 0.8	0.8 0.9 0.9		0.8 0.9 0.9		0.6 0.7 0.8	8 8 8

FIGURE 09. *Overlap time factor for B_{ij} represented by triangular functions (adapted from Shi and Blomquist, 2012)*

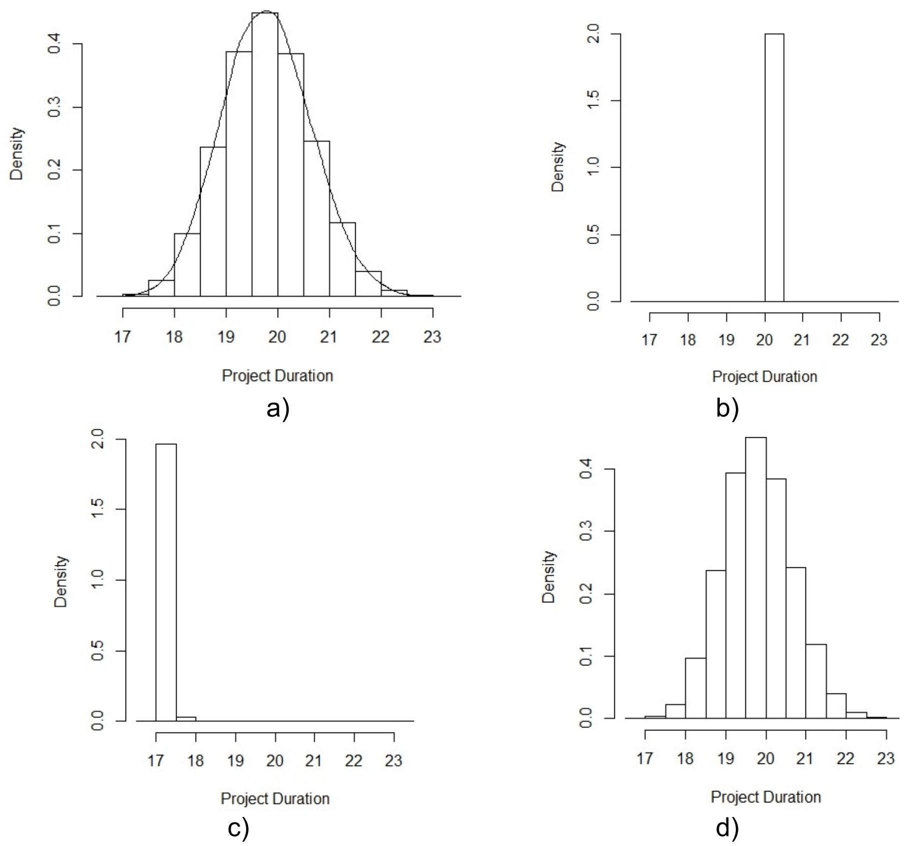


FIGURE 10. *Histogram for project duration. a) Uncertainty in all input variables; b) fifteen most influent input variables fixed at their m values; c) fifteen most influential input variables fixed at l values; and d) fifty least influential input variables fixed at their m values*

The Sobol-Jansen method was applied to identify the most influential input variables. The total cost was 2,010,000 model calls because there are 65 input variables, and a sample size of 30,000 was used. Despite the large number of model calls, only a few minutes were needed in a notebook with an Intel® Core i7 processor because the model is very simple. **Figure 12** shows the total and first-order Sobol indices for the 20 most influential input variables. The other 45 input variables have Sobol indices equal to zero. There are nine and 12 input variables with total Sobol indexes greater than 0.01 and 0.001, respectively. The activity durations of activities *A*, *D*, and *J* and the time factors C_{JG}, B_{JG}, C_{GD}, C_{RJ}, B_{RJ}, and B_{DA} are the most influential input variables.

Fig. 10. Histogram for project duration. **a)** Uncertainty in all input variables; **b)** fifteen most influent input variables fixed at their m values; **c)** fifteen most influential input variables fixed at l values; and **d)** fifty least influential input variables fixed at their m values

To demonstrate that these variables are the most influential, three scenarios are analyzed. In scenario 1, the 15 most influential input variables are fixed at their m values, meaning that these variables can be controlled at that value. In scenario 2, the 15 most influential input variables are fixed at their l values. Finally, in scenario 3, it is assumed that all variables, other than the first 15, can be fixed at their m values, meaning that the uncertainty of these variables can be eliminated from the uncertainty analysis because they are not influential. Monte Carlo simulation was used with 100,000 model calls.

Figure 10 shows the histogram of the project duration for these scenarios. **Figures 10b** (scenario 1) and **10c** (scenario 2) show that the uncertainty in project duration can be eliminated if the values of the 15 most influential input variables are fixed. This result indicates that these variables are responsible for the uncertainty in the project duration and that the uncertainty in the other 50 input variables does not affect the project duration. Conversely, **Figure 10d** (scenario 3) shows that if the uncertainty in the last 50 input variables is eliminated, the uncertainty in the project duration is almost equal to the uncertainty in the project duration if uncertainty is considered in all variables (**Figure 10a**).

Figure 11 shows similar results for the early start and early finish uncertainty. **Figures 11b** and **11e** show that the uncertainties of the key activities are eliminated if the uncertainties of the 15 most influential input variables are eliminated. Conversely, **Figures 11c** and **11f** show that the uncertainty in the key activities is very similar if all input variables have uncertainty (**Figures 11a** and **11d**) or the uncertainty of the last 50 input variables is eliminated.

In conclusion, it is demonstrated by exhaustion (from the 100,000 Monte Carlo simulation) that **1)** the control of the uncertainty of the most influential input variables reduces the uncertainty in the key activities and the project duration, and **2)** the uncertainty analysis can be performed using only the most influential input variables.

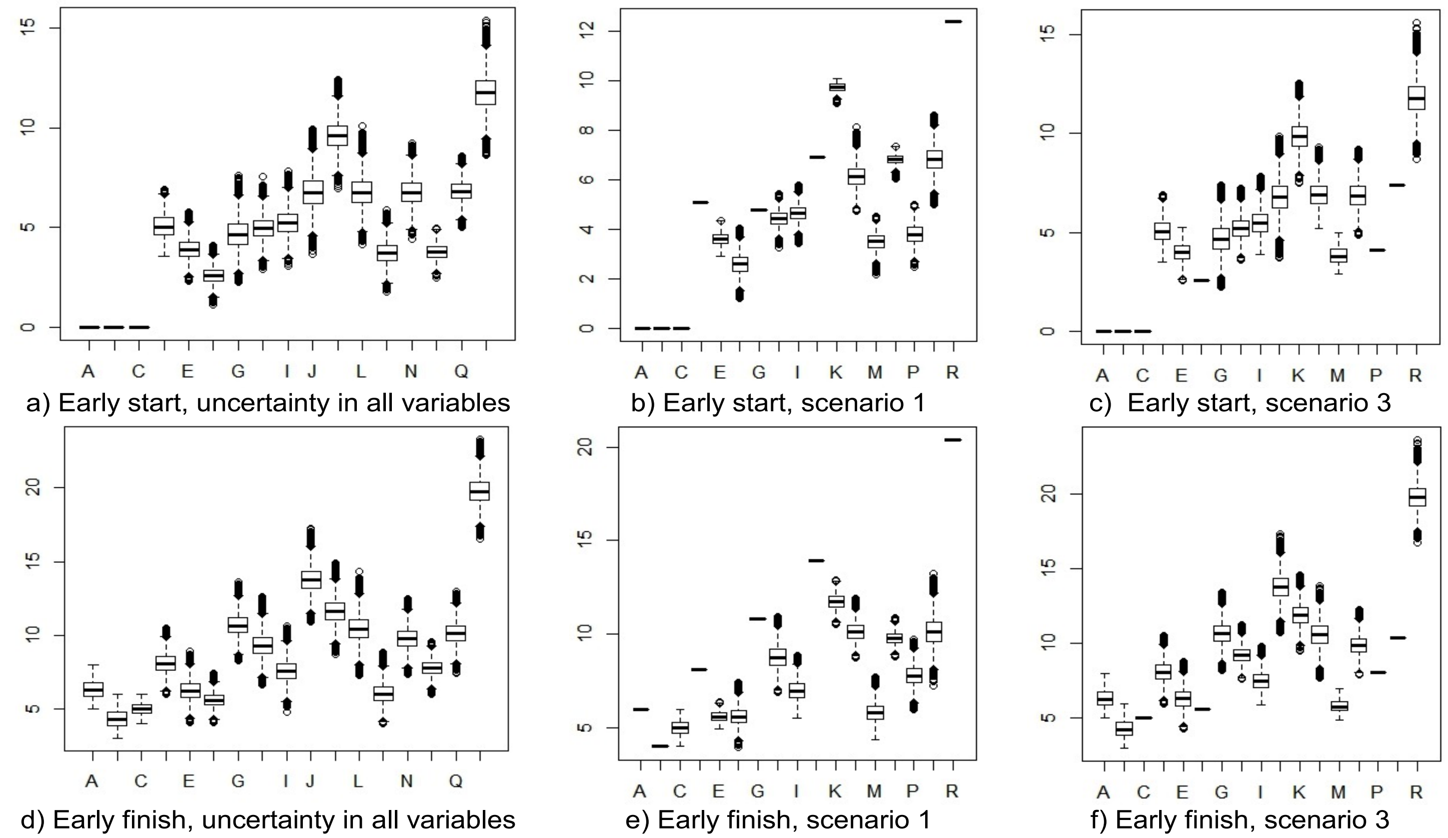


FIGURE 11. *Box plot for early start and early finish in the case study*

5. Conclusion and final comments

Uncertainty and sensitivity analyses have been applied to project duration using DSM-based scheduling, as proposed by Maheswari and Varghese. The characterization of the uncertainty is a key component of the uncertainty analysis of project duration. The type of distribution function does not play as important a role as the magnitude of the uncertainty. The project duration uncertainty follows a normal distribution if the magnitudes of the uncertainty in the input variables are moderate and is independent of whether the input variables follow uniform, normal, or log-normal distributions. Random sampling is sufficient because the model is simple and has low computational costs. A cumulative function for project duration is a good way to present the uncertainty to determine the probability that the project duration is equal to or less than a certain time. A box plot is a good way to present the uncertainty in the early start and the early finish of all (or groups of) activities.

The Sobol method has been proposed to be used for identification of the input variable uncertainty that is responsible for the uncertainty in project duration. The use of first-order and total Sobol sensitivity indices is a better option than using the second- or third-order Sobol sensitivity indices. The Sobol method has been shown to be adequate in the ranking and selection of the input variables. This ranking and selection of activities can be used to concentrate efforts and resources on a few activities to define their uncertainty or to control the duration of such key activities. The control or reduction of the uncertainty of the key activities durations was demonstrated to reduce uncertainty in the project duration. If resources are limited, approximate uncertainty can be assigned to the duration of activities and time factors. After

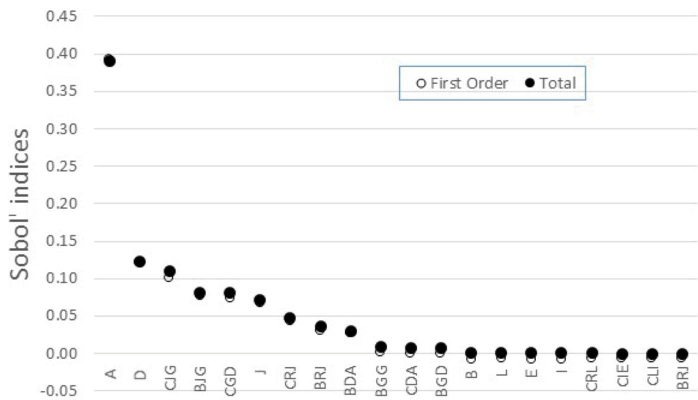


FIGURE 12. *Estimation of the Sobol-Jansen indices for the case study of the 20 most influent input variables*

the keys input variables are identified, resources can be allocated to estimate the uncertainty of the key input variables. In addition, the uncertainty analysis can be performed using only the influential input variables.

Project scheduling includes several other aspects to be considered along with the sequence of activities. These aspects include resource allocation, budget setting, and organizational structure, among several others. From that perspective, identifying insignificant input variables can help simplify the problem by eliminating the uncertainty of those variables. Moreover, a project can contain many activities, perhaps thousands of activities, that may hinder the implementation of GSA. In such cases, it is advisable to identify milestones and divide the project into subprojects. Then, GSA can be applied to the project considering that each sub-project is an activity, and GSA is applied to each sub-project individually.

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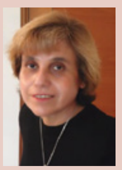

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