DOI NUMBER: 10.19255/JMPM01305

PROJECT DURATION

KEYWORDS
Regionalized sensitivity analysis • Dependency structure matrix • Monte Carlo Filtering • Standardized regression coefficient • Project scheduling.

A Method for IDENTIFICATION OF CRITICAL SCHEDULING DECISIONS

EDELMIRA D. GÁLVEZ
• Department of Mining and Metallurgy Engineering
• Universidad Católica del Norte, Antofagasta - Chile
• egalvez@ucn.cl

JOAQUIN B. ORDIERES
• Department of Industrial Engineering, Business Administration and Statistics
• Universidad Politécnica de Madrid
• j.ordieres@upm.es

SALVADOR F. CAPUZ-RIZO
• Department of Engineering Projects
• Universitat Politècnica de València, Valencia - Spain
• gscapuz@dpi.upv.es

• ABSTRACT •

This paper presents a method for determining and controlling variables which are critical to the desired duration of a project. The proposed method consists of four stages: 1) a Monte Carlo simulation of project scheduling using a dependency structure matrix (DSM), 2) the reduction of the number of input variables using an index based on the standardized regression coefficient, 3) the determination and regionalization of critical variables using a modified Monte Carlo Filtering (mMCF) method, and 4) an evaluation of risk of the project duration under the regionalized conditions. The proposed method was applied to three case studies. Results show that the method could be helpful in scheduling projects to obtain the desired duration under uncertainty, identifying critical variables, and regionalizing the critical input variables.

1. INTRODUCTION

Projects in numerous industries are subject to many uncertainties attributable to several possible causes. For example, the durations of project activities are dependent on uncertain factors, and activity durations are therefore random rather than deterministic variables (Carr, 1979). This uncertainty can be attributed to an absence of knowledge about their values under definite circumstances (epistemic uncertainty) or the fact that the variable exhibits naturally random behavior (stochastic uncertainty). This uncertainty may affect the completion date of a project, which can translate into higher costs. For this reason, several studies have analyzed uncertainty in project planning (Herroelen and Leus, 2005). Most studies have performed uncertainty analysis, i.e., analyzing the uncertainty behavior in project duration as a result of uncertainty in the input variables. However, few studies have performed sensitivity analysis, i.e., identifying input variables that are responsible for the uncertainty in the output variable.

The dependency structure matrix (DSM) has been used to represent projects, especially in cases in which there is interdependence between activities (Browning, 2001). The DSM has several applications in project management. Maheswari and Varghese (2005) developed a methodology for determining project duration using the DSM. They included projects with communication times between activities and projects with overlap between activities. More recently, Srour et al. (2013) applied the DSM to the planning of construction projects, including projects with overlap. However, the models of Maheswari and Varghese (2005) and the extension of Srour et al. (2013) are deterministic, i.e., they do not consider the uncertainty in the variables, which limits their application to real situations and construction projects in particular. Additionally, project risk management requires assessments of project duration and activity criticality (Yang, 2007).

The introduction of overlap in the DSM is important because a scheduler can accelerate a construction project by setting up overlapping activities. Activities are considered to be overlapping if two activities that are normally executed in sequence are performed in parallel, with the successor starting before the end of the predecessor. Recently, Lim et al. (2014) presented a method that identifies an optimal overlap rate between critical activities to provide a time-cost trade-off analysis, hence reducing completion time and cost without allocating additional resources. It is important for the contractor to minimize the project completion time. There is a pressing need for the contractor to recognize and achieve this objective. In response, researchers have proposed various methods that can be classified into optimization-based scheduling and concurrency-based scheduling.

Gálvez et al. (2012) adjusted the methodology proposed by Maheswari and Varghese to include uncertainty in the input variables. All variables were represented by grey numbers, which allowed for the calculation of project duration as a grey number. Shi and Blomquist (2012) later introduced fuzzy numbers into the Maheswari and Varghese procedure to represent uncertainty in project planning. Recently, Gálvez et al. (2015a) represented the input variables in the Maheswari and Varghese model using distribution functions and studied the corresponding project duration behavior using Monte Carlo simulations. In that study, it was observed that project duration tends to have a normal distribution, independent of the distribution function used to represent the input variables. Moreover, it was observed that the grey numbers tend to overestimate the uncertainty in project duration. All of these studies pertain to uncertainty analysis; that is, they involve determining the effect of the uncertainty of the input variables on the uncertainty of output variables.

Only few studies have been published on the application of global sensitivity analysis (GSA) to project planning and scheduling. GSA seeks to identify the input variables that are responsible (influential) for the uncertainty of output variables. Therefore, GSA can be very useful in project management in the identification of influential and non-influential input variables. This information can be used to reduce the size of models (by removing non-influential input variables) or to control the output variable uncertainty (by controlling influential input variables), as demonstrated in several science and engineering studies. For example, some studies have involved model simplification and efficient calibration in the water productivity model AquaCrop (Vanuytrecht et al., 2014), determination of rate coefficients of interest in combustion (Shannon et al., 2015), interpretation of biological experiment results (Jarret et al., 2015), analysis of urban water quality modelling (Vanrolleghem et al., 2015), design of mineral processes (Sepúlveda et al., 2014), studies in probabilistic engineering design (Sathyanarayanamurthy and Chinnam, 2009) and understanding the main sources of uncertainty affecting the risk of CO_2 escape from the geological carbon storage (Gonzalez-Nicolas et al., 2015). Gálvez et al. (2015b), Gálvez and Capuz-Rizo (2016), and Gálvez et al. (2017) showed that GSA can be used to identify which input variable uncertainties (project duration and overlap factors) are influential/non-influential in project duration uncertainty using the DSM. The authors used this information to reduce project duration uncertainty by controlling the uncertainty of the influential input variables. Several GSA methods were compared and analyzed as a potential tool in project scheduling using the DSM.

A different GSA problem is the identification of the comparative importance (critical, important, insignificant) of input variable uncertainty in determining a specific behavior of the output variables. This type of problem is known as regionalized sensitivity analysis because it involves searching for input variables that are responsible for the output variables being in a region of interest (Saltelli et al., 2004). Monte Carlo Filtering (MCF) was developed for this purpose (Spear and Hornberger, 1980). The method consists in Monte Carlo simulation and in classifying the model output as "behavior" or "non-behavior". The simulation results are used to determine whether the distribution functions of each input variable, in the "behavior" and "non-behavior" data set, are identical (insignificant) or different (critical). MCF was used by Brockmann and Morgenroth (2010) to identify operating conditions that result in a desired biofilm system behavior. However, MCF yields better results for small models, typically those with fewer than 20 input variables (Saltelli, 2004). For this reason, Lucay et al. (2015) recently proposed reducing model size using the Sobol´ method (Sobol', 1993) to eliminate non-influential input variables before using MCF. The authors applied the procedure for the identification and regionalization of critical variables in chemical processes. The identification of the critical input variable uncertainty is very important when determining a specific behavior of the project duration. To define project goals, it is necessary to determine the range of feasible values of project duration, and knowing which variables are critical to obtain a desired project duration allows to control these variables in order to reach the defined goal.

The objective of this research is the development of a method for identifying the input variables responsible for a certain behavior of the project duration. It is also shown that controlling or regionalizing these variables may increase the likelihood of the project duration being close to the desired value. This paper is divided in five sections, the first of which is the introduction. The second section presents the proposed methodology. This section is divided into four sub-sections, one for each stage of the method. Then, in the fourth section, three case studies are presented to validate and exemplify the methodology. Finally, conclusions are presented in section five.

2. PROPOSED METHODOLOGY

As indicated above, the methodology consists of four stages: 1) a Monte Carlo simulation of project scheduling using the DSM, 2) the reduction of the number of input variables using an index based on the standardized regression coefficient, 3) the determination and regionalization of critical variables using a modified Monte Carlo Filtering method, and 4) an evaluation of the risk associated with project duration under the regionalized conditions.

--- 2.1 Simulation of project scheduling ---

This stage includes the uncertainty analysis based on a Monte Carlo simulation of project scheduling. This uncertainty analysis allows defining the desired (and undesired) behavior of the project duration. In this study, the method developed by Maheswari and Varghese (2005) and extended by Gálvez et al. (2015a) for the estimation of project duration with and without natural overlap between activities is used. However, other methods can be utilized to simulate project duration. In the method of Maheswari and Varghese, the dependence between activities, activity duration, and overlap time factors are used to estimate the project duration. Here, the same models are used, but uncertainty is considered in all input variables. This method can be applied to both the traditional (sequential: an activity starts once its predecessors are completed) method and the phased method (some overlap occurs between pairs of activities); however, the phased method is emphasized in this study because problems that require this method are more complex and several strategies can be used to control project duration.

The project duration has been chosen as the output variable, but other variables such as early start, late start, early finish, late finish, and slack time can be considered as well. Therefore, the method of Maheswari and Varghese can be extended to include the calculation of late start, late finish, and slack time, as shown in appendix A.

The Maheswari and Varghese method uses the DSM to calculate project duration. The DSM is a square matrix containing a list of activities in the rows and columns in the same order. The order of activities in the rows and columns in the matrix indicates the sequence of execution. Values along the diagonal are the durations of the activities, and values along the off-diagonal cells indicate that these activities are information predecessors, with activity inputs in the rows and activity outputs in the columns.

For the sequential method, Gálvez et al. (2015a) proposed the following equations for the calculation of early start (ES), early finish (EF) and project duration (P) using the DSM:

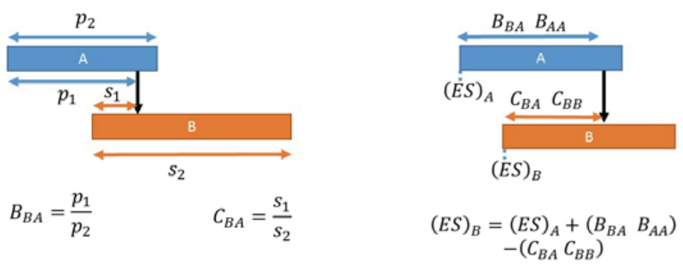
$$(EF)_i = (ES)_i + A_{ii} \qquad 0< i \le n \qquad (1)$$

$$(ES)_j = \text{Max}\,[(EF)_i\,] \qquad 0< i \le n,\ 0< j \le n \qquad (2)$$

$$P = \text{Max}\,[(EF)_i] \qquad 0< i \le n \qquad (3)$$

Where *j* is the current activity (the activities are selected following the order given by the DSM), *i* indicates all of the immediate predecessors of *j*, A_{ii} are the diagonal values of the DSM representing the duration of activity *i*, and *n* is the number of activities in the project.

For natural overlap methods of project scheduling, Maheswari and Varghese introduced time factors. Two times factors are used: the time factor related to receiving information for the successor activity (given by the off-diagonal matrix values of matrix B_{ji}) and the time factor related to sending the information from the predecessor activity (represented by the off-diagonal values of matrix C_{ji}). See **Fig. 1** for a graphic definition of B_{ji} and C_{ji}. The overlap considered by these factors is natural overlap (in contrast to forced overlap), and therefore, rework is not included. Values of 1 and 0 in B_{ji} and C_{ji}, respectively, indicate that overlap is not possible or allowed between these activities (and equations 4 to 6 became the equations 1 to 3).



Graphic definition of B_{ji} and C_{ji}. Graphic description of equation 4.

FIGURE 01. Graphic description of natural overlap matrix and equations.

The natural overlap project duration is estimated as follows (Maheswari and Varghese, 2006):

$$(ES)_j = \text{Max}\,[(ES)_i + B_{ji}\ B_{ii} - C_{ji}\ C_{jj}] \qquad 0< i \le n,\ 0< j \le n \qquad (4)$$

$$(EF)_i = (ES)_i + B_{ii} \qquad 0< i \le n \qquad (5)$$

$$\text{natural overlap project duration} = \text{Max}\,[(EF)_i] \qquad 0< i \le n \qquad (6)$$

B_{ii} and C_{ii} are the diagonal values of the DSM (duration of activity). **Fig. 1** explains the equation 4. A short calculation example is presented in **Appendix A**.

--- 2.2 Reduction of input variables ---

The quality of MCF results increases with few input variables. If the schedule problem involves more than 20 uncertain input variables, then it will be necessary to reduce the number of uncertain variables. Gálvez and Capuz-Rizo (2016) assessed GSA methods for the identification of influential and non-influential input variables in project scheduling using the DSM. They included scatterplots, partial correlation coefficient, partial rank correlation coefficient, standardized regression coefficient (SRC), standardized rank regression coefficient, Morris (1991), and Sobol' (1993) methods. Based on the results obtained by Gálvez and Capuz-Rizo, (2016) the SRC, Morris and Sobol' methods can be used to identify and sort input factors that affect the uncertainty in project duration. However, the Sobol' method features a higher computational cost and is more difficult to implement. The SRC and Morris methods also allow for identifying the direction of the correlation between project duration and a given input factor. However, Morris method does not consider the form (only the range) of the input variable distribution function. On the other hand, the SRC method has a low computational cost, it is easy to implement, and it considers the form and range of the input variable distribution function. Therefore, the SRC method is used here to reduce the input variables. Please note that although the SRC, Morris, and Sobol' methods allow for sensitivity analysis over the full range of variable uncertainties, they do not allow for studying or identifying regionalized behaviors.

The foundation of the SRC method is performing a Monte Carlo simulation of the model. The number of simulations must be sufficiently high to obtain stable evaluations of the coefficients, which can be accomplished by trial and error. The space of activity durations and overlap factors (x_j variables) is explored by random sampling from the distribution function of these variables. A multivariate linear model relating the project duration (*y* variable) to the input variables (x_j) is then fitted. This is

$$\hat{y} = b_0 + \sum\nolimits^{n}_{(j=1)} b_j\ x_j \qquad (7)$$

The linear regression coefficients (b_j) are not useful for sensitive analysis in our case because activity duration and overlap factors have different units and because they do not integrate information on the distribution allocated to these variables. The SRC_j valued are obtained by normalizing the linear regression coefficient (b_j) associated with the input variable x_j:

$$SRC_j = b_j \frac{s_j}{s} \qquad (8)$$

where S_j and S are the sample standard deviations for the input variable x_j and output variable y, respectively.

These SRC_j values provide a valid measure of sensitivity if the coefficient of determination, R^2, is greater than 0.7 because R^2 is a measure of the degree to which the regression can match the observed data. Specifically, a value of R^2 close to 1 indicates that the regression model is effective in accounting for the uncertainty of *y*, and a value of R^2 close to 0 indicates that the regression model is not successful in representing the uncertainty of *y*. On the other hand, based on the work of Gálvez and Capuz-Rizo (2016), the SRC_j values provide a valid measure for GSA in project scheduling using DSM.

The absolute value of SRC delivers a measure of variable importance. The uncertainty of a variable has a greater effect on the uncertainty in the project duration (the variable is more influential) if its absolute value of SRC is greater than the absolute value of another variable. The SRC also provides a measure of the direction of the relationship between two variables. A positive value indicates that both *y* and x_j are decreasing or increasing together, whereas a negative value means that the input and output factors tend to move in opposite directions. SRC can be used to reduce model size by classifying input variables as influential or non-influential. The non-influential variables are then fixed to nominal values.

To reduce the number of input variables, the following steps are proposed. First, the SRC_j is determined for all the input variables. Then, the normalized absolute values of SRC are calculated using the following equation:

$$\text{Normalized absolute value of } SRC_j = \frac{|SRC_j\,|}{\sum_k |SRC_k\,|} \qquad (9)$$

Note that the summation of the normalized absolute SRC is equal to 1. Then, the normalized absolute value of SRC is assorted from the higher to the lower value. The larger the value of normalized absolute SRC is, the greater the effect of uncertainty of that variable on the uncertainty of the project duration. Next, the cumulative of normalized absolute SRC_j (NS_j) is determined by using:

$$NS_j = \sum\nolimits^{j}_{(i=1)} \text{ Normalized absolute value of } SRC_i \qquad (10)$$

Where the normalized absolute value of activity *i* is greater than the value of activity *j*. The NS_j values indicate that the first *j* input variables are responsible for 100 NS_j % of the uncertainty of the project duration. A reduction of the number of input variables is then proposed by considering the variables that are responsible for 97% of the uncertainty of the project duration.

--- 2.3 Determination and regionalization of critical variables ---

The determination and regionalization of critical variables involves identifying the input variables (critical variables) that are responsible for a specific behavior of the project duration. In the identification of these critical variables, only the variables identified as influential in the previous stage are considered. All non-influential variables are fixed to their nominal value. To determine and regionalize these critical variables, a modified MCF method is developed. In the MCF method (Spear and Hornberger, 1980), the following criterion is used for the output variables: the results are divided into two subsets, one with the values of the input variables that are responsible for a desired behavior of the model output (set R) and another set with the values of the input variables that provide the unwanted behavior (set R^-, i.e., for the X_i variable of a particular model, the number of elements of X_i that are part of set R is *n* (# (X_i/R)=*n*) and the number of elements of X_i that are part of set R^- is *m* (# (X_i/R^-)=*m*) such that *n*+*m*=*N*, where *N* is the total number of simulations, i=1,…,*k*, and *k* is the total number of input variables in the model. However, this division in two subsets usually generates subsets of unequal number of elements that reduces the statistical power of the method. In addition, Spear et al. (1994) indicated that the fraction of R is hardly larger than 5 % of the total number of simulations for models with input variables over 20, implying a lack in statistical power. Therefore, in this work the desired behavior of the model output (set R) and the unwanted behavior (set R^-) are selected so that *n* and *m* are roughly equal (and *n*+*m*<*N*), and *n* and m are selected so that both values are greater than 10% of the total simulations (n>0.1 N and m>0.1 N). This is then called the modified Monte Carlo Filtering method (mMCF).

If the input variable X_i is a key or critical variable to the behavior of the model output, the performance of its density functions in R and $\bar{R}$, i.e., $f_R(X_i/R)$ and $f_{\bar{R}}(X_i/\bar{R})$, will be significantly different. On the other hand, if the density functions are not significantly different, then input variable X_i is not a critical variable (insignificant) to the behavior of the model output. This procedure is carried out for each input variable. The density functions of two samples can be compared using the Kolmogorov-Smirnov (KS) test. The KS statistic quantifies the distance between the distribution functions of two samples. The null distribution of this statistic is calculated under the null hypothesis, H_o, that the two density functions $f_R(X_i/R)$ and $f_{\bar{R}}(X_i/\bar{R})$ are identical and the alternative hypothesis, H_1, that the two density functions are dissimilar:

H_o: $f(X_i/R) = f(X_i/\bar{R})$

H_1: $f(X_i/R) \neq f(X_i/\bar{R})$

The test is defined as follows:

$D_i = \sup \| F_R(X_i/R) - F_{\bar{R}}(X_i/\bar{R}) \|$

where F is the cumulative probability function and sup represents the supremum of $\|F_R(X_i/R) - F_{\bar{R}}(X_i/\bar{R})\|$, which is greater than or equal to all values of $\| F_R(X_i/R) - F_{\bar{R}}(X_i/\bar{R})\|$.

The decision criterion is as follows: If $D_i \leq D_\alpha$, then accept H_o , and if $D_i > D_\alpha$, then reject H_o , where P(reject H_o / H_o is true)=P($D_i > D_\alpha$) = α.

This type of decision is typically made based on p-value. The p-value is the probability that, if the null hypothesis were true, the KS D_i statistic would be as large as or larger than the observed value. That is, p-value =P(D_i>D_α / H_o is true). Therefore, the smaller the p-value is, the more evidence for rejecting the null hypothesis there is. If the null hypothesis is rejected, then the alternative hypothesis is accepted. Usually, if p ≤ 0.05 the null hypothesis is rejected, but here the p-values proposed by Saltelli et al. (2014) are used to determine whether the input variable is critical or not (reject or accept the null hypothesis): If p-values < 0.01, then the variable is critical; if 0.01 ≤ p-values ≤0.1, then the variable is important, and if p-values ≥ 0.1, the variable is insignificant.

Once the critical variables are identified, the input variable values that produce the desired performance in the model output (set R) can be analyzed to regionalize or identify target values for the critical variables.

2.4. Evaluation of risk of the project duration under the regionalized conditions.

Our objective is to identify the values of the activity duration and/or overlap factor that allows the desired behavior of the project duration to be obtained. This objective can be met by observing the behavior or values of the critical variables in the set R. After defining these values, the risk of obtaining the expected values of project duration must be analyzed, which can be accomplished by Monte Carlo simulation with the critical variables in the defined values and the other variables with the original uncertainties. One clearly desired behavior of the project duration is to compress the schedule. In the literature, three strategies have been used to compress a schedule (Carr, 1979). Fig. 2a illustrates a time-cost trade-off analysis that makes use of optimization-based scheduling. This strategy reduces project duration by assigning additional resources and/or cost. Fig. 2b demonstrates concurrency-based scheduling, which reduces duration by overlapping predecessor and successor activities without assigning additional resources. Both strategies (accelerating and overlapping activities) can be used simultaneously, as shown in Fig. 2c. By performing project scheduling using the DSM, mMCF and regionalization of the input variables, all previous strategies can be used to compress the schedule. If the acceleration of activities is desired, then uncertainty in activity duration must be considered, and equations 1 to 3 are used to model project duration. On the other hand, if the overlapping of activities is to be considered, uncertainty in the overlap factors, deterministic values for activity duration, and equations 4 to 6 must be used. Finally, if both accelerating and overlapping activities are to be considered, the uncertainties in all input variables are considered jointly using equations 4 to 6.

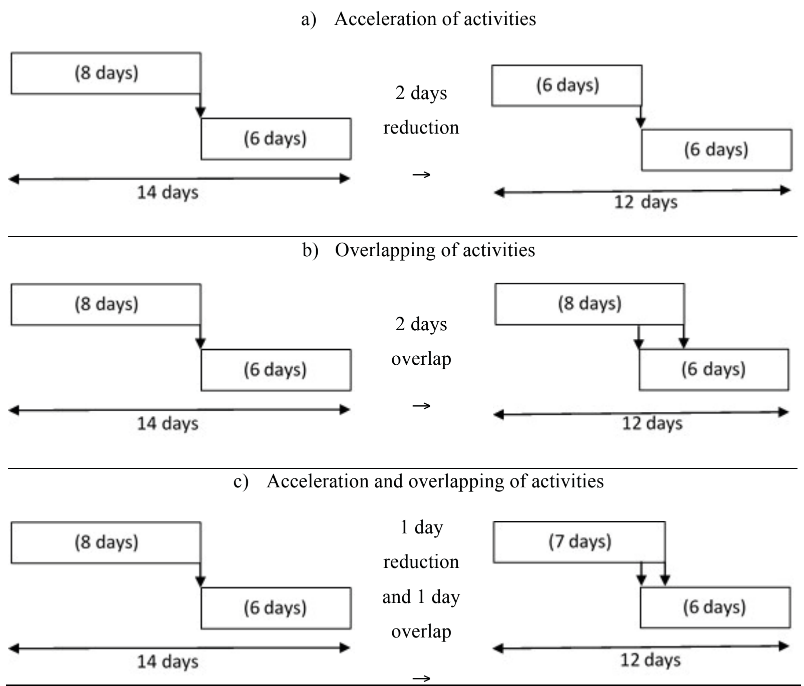


FIGURE 02. Strategies for compressing a schedule

As described in the following section, mMCF and Monte Carlo simulation were implemented in the R software (R Core Team, 2013) and SRC was implemented in the R 'sensitivity' package (Pujol et al., 2014). The Monte Carlo simulation, the SRC, and the mMCF need to define a sample size. There is no rule for such definition; therefore, in this work the number of simulations required was determined so the results would not change with a variation in the random sample chosen. This was performed by trial and error; therefore, its size may not be optimal (minimum size).

3. CASE STUDIES

The method is applied to three case studies. The first case study schedules a project with six activities and 20 input variables, as presented by Gálvez et al. (2015b). The focus of this example is the regionalization of the critical variables, and the reduction of the number of input variables is not considered. The second case study considers a project with ten activities and 40 input variables, as presented by Maheswari and Varghese (2005). The focus of this example is the reduction of input variables. The third case study considers a road pavement project with 26 activities presented by Yang (2007). In the first two case studies, accelerating and overlapping activities are involved, and in the third case study only an acceleration activity is involved. In the first two case studies only uniform distribution functions are considered, but in the third case study uniform and discrete distribution functions are considered for the input variables. These types of distribution functions are used because they represent alternative values of these input variables to obtain the target project duration.

--- 3.1 Case 1: Gálvez et al. case study ---

This example was presented by Gálvez et al. (2015b) but some changes were introduced in the input factor to better describe the method proposed here. The project has six activities, and the B_{ij} and C_{ij} matrices are presented in **Fig. 3**. In **Fig. 3**, U(a,b) represents a uniform distribution between the lower bound a and the upper bound b. Values along the diagonal (B_{ii} and C_{ii}) are the activity durations, and values along the off-diagonal (B_{ij} and C_{ij}) are the overlap time factors. For example, U(3,5) in B_{AA} indicates that activity A has an uncertain duration of U(3,5); U(0.8,1.0) in B_{CA} denotes that A can drive the required information to C at the end of U(0.8,1.0) times its duration; and U(0.0,0.2) in C_{CA} denotes that activity C needs information from A, but not at the beginning of activity C, instead at U(0.0,0.2) of the time of its duration. Although this example has only six activities, it is not a simple example because the interdependence between the activities and the level of overlap can affect the behavior and the effect of each activity on the project duration. Based on the mean values of the activity durations and mean values of the factor times, the natural overlap project duration is estimated to be 18 days (**Fig. 4**). Additionally, it can be shown (see **Appendix A**) that all activities, with the exception of activity C, are part of the critical path, but it is not possible to known which overlap factors are also important for the critical path. The total float time of activity C is 0.3. Because the activity duration of C is U(4,6), it can be inferred that C can also be critical because of its uncertainty. The objective is to identify the critical input variables that control the project duration between given values. For all the Monte Carlo simulations and for all the mMCFs a sample of 10,000 and 6,000 points were used respectively. These samples, as indicated before, were determined by trial and error so the results would not change with a change in the random sample chosen.

The first step of the method is the simulation of project scheduling using the DSM; this process involves equations 4 to 6 and the information in **Fig. 3**. Monte Carlo simulation for the project duration yields a minimum value of 14.08, a first quartile of 17.87, a mean of 18.70, a third quartile of 19.51, and a maximum of 23.80. Thus, given the potential range of values for activity duration and overlap factors (uniform distribution), the project duration can take values between 14.08 and 23.8. The mean value (18.7) is greater than the value estimated using the mean values of activity durations and factor times (18.0) because the early start of activities D and F is a function of the behavior of activities B-C, and D-E, and their overlap.

The second step of the method is the reduction in the number of input variables using GSA methods. However, this problem features 20 input variables (6 activity durations and 14 overlap time factors), which is why reducing the number of input variables may not be necessary and therefore not included here.

The third and fourth steps depend on the aim defined for the project duration. Here,

	A	B	C	D	E	F
A	U(3,5)					
B	U(0.7,1.0)	U(5,7)				
C	U(0.8,1.0)		U(4,6)			
D		U(0.8,1.0)	U(0.8,1.0)	U(6,8)		
E		U(0.8,1.0)			U(6,8)	
F				U(0.8,1.0)	U(0.8,1.0)	U(4,6)

a) Time factor of processor activities (B_{ij})

	A	B	C	D	E	F
A	U(3,5)					
B	U(0.0,0.3)	U(5,7)				
C	U(0.0,0.2)		U(4,6)			
D		U(0.0,0.2)	U(0.0,0.2)	U(6,8)		
E		U(0.0,0.2)			U(6,8)	
F				U(0.0,0.2)	U(0.0,0.2)	U(4,6)

b) Time factor of receiving information (C_{ij})

FIGURE 03. DSM showing the uniform distribution of duration of activities and time factors of information transfer between activities.

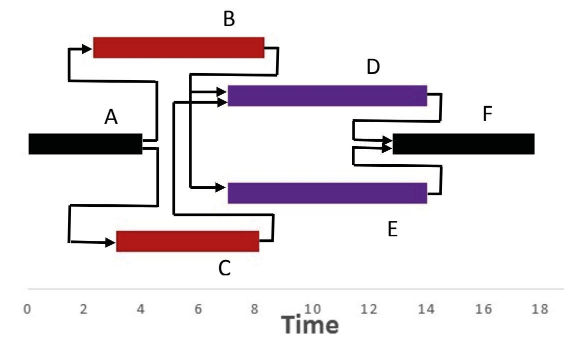


FIGURE 04. Project scheduling with mean values of the input variables.

two scenarios are considered for illustration.

Scenario 1. Reducing the risk of late delivery.

The third step in the method is the determination and regionalization of critical variables by the modified Monte Carlo filtering. All 20 input variables are considered to determine which have the greatest influence on whether the project duration exceeds 19.51 (third quartile). This procedure reduces the risk of late delivery of the project. Because it is desirable to avoid a project duration equal to or above 19.51, sets R and $\bar{R}$ are defined as follows:

R={x / 18.70≤ project duration(x) <19.51, x ∈E}

$\bar{R}$={x / 19.51≤ project duration(x), x ∈E}

where project duration(x) is the project duration at point x and E is a space formed by the uncertainties of 20 twenty input variables considered. In the set R, the lower bound is the mean value of the project duration. mMCF was applied by taking the uncertainty ranges presented in **Fig. 2** and considering project duration as the output variable. The density functions of sets R and $\bar{R}$ associated with each variable were compared using the Kolmogorov-Smirnov test. The results are displayed in **Table 1**, which shows that there are 13 critical variables that affect the desired behavior of the project duration, 4 insignificant variables that do not affect the desired behavior of the project duration, and 3 important variables that may affect the desired behavior of the project duration. The important variables are in an intermediate position and therefore can or cannot be considered critical. If these variables are not considered, the risk of late delivery may increase. One way to consider this situation is by analyzing the cumulative probability functions of sets R and $\bar{R}$. These cumulative probability functions are shown in Appendix B. It can be observed that the difference between the two cumulative probability functions for all important variables is not significant, and we will therefore consider only the critical variables.

The mean values of the critical variables obtained in set R can be used to obtain the desired behavior of the project duration (18.70≤ project duration <19.51). These values are B_{AA} = 4.07, B_{BB} = 6.06, B_{DD} = 7.07, B_{EE} = 7.02, B_{FF} = 5.10, B_{BA} = 0.856, B_{DB} = 0.902, B_{EB} = 0.903, B_{FD} = 0.905, B_{FE} = 0.902, C_{BA} = 0.141, C_{EB} = 0.098, and C_{FD} = 0.098. The non-critical variables can take any values in the uniform distribution that represent their uncertainties. Thus, the critical variables must be controlled to have values close to the mean values. If the values are the expected ones, they are close to the mean values of the activities in the critical path; however, there are surprise C_{DB} and C_{FE} variables that are not critical variables. **Fig. 5** highlights the critical variables for **scenario 1**; note that the arrows in the arc do not denote direction but the overlap factors that are critical.

Additionally, the regionalization of the critical variables can be analyzed to obtain the

Scenario 1				Scenario 2			
Input factor	D	p-values	Input factor is	Input factor	D	p-values	Input factor is
B_{AA}	0.2121	0.0000	critical	B_{AA}	0.3892	0.0000	critical
B_{BB}	0.1268	0.0000	critical	B_{BB}	0.1943	0.0059	critical
B_{CC}	0.0263	0.5230	insignificant	B_{CC}	0.2318	0.0005	critical
B_{DD}	0.1003	0.0000	critical	B_{DD}	0.1346	0.1221	insignificant
B_{EE}	0.0759	0.0000	critical	B_{EE}	0.1341	0.1246	insignificant
B_{FF}	0.1885	0.0000	critical	B_{FF}	0.3159	0.0000	critical
B_{BA}	0.0989	0.0000	critical	B_{BA}	0.1839	0.0108	important
B_{CA}	0.0361	0.1656	insignificant	B_{CA}	0.1448	0.0786	important
B_{DB}	0.0542	0.0072	critical	B_{DB}	0.2000	0.0042	critical
B_{DC}	0.0176	0.9290	insignificant	B_{DC}	0.0799	0.7083	insignificant
B_{EB}	0.0614	0.0015	critical	B_{EB}	0.1089	0.3190	insignificant
B_{FD}	0.0815	0.0000	critical	B_{FD}	0.2977	0.0000	critical
B_{FE}	0.0785	0.0000	critical	B_{FE}	0.1685	0.0249	important
C_{BA}	0.1461	0.0000	critical	C_{BA}	0.2152	0.0016	Critical
C_{CA}	0.0183	0.9070	insignificant	C_{CA}	0.1469	0.0714	Important
C_{DB}	0.0430	0.0585	important	C_{DB}	0.1456	0.0756	Important
C_{DC}	0.0520	0.0114	important	C_{DC}	0.1878	0.0086	Critical
C_{EB}	0.0569	0.0041	critical	C_{EB}	0.0855	0.6255	insignificant
C_{FD}	0.1133	0.0000	critical	C_{FD}	0.1353	0.1186	insignificant
C_{FE}	0.0397	0.0985	important	C_{FE}	0.0804	0.7012	insignificant

TABLE 01. Modified Monte Carlo filtering results and classification of input variables for scenarios 1 and 2

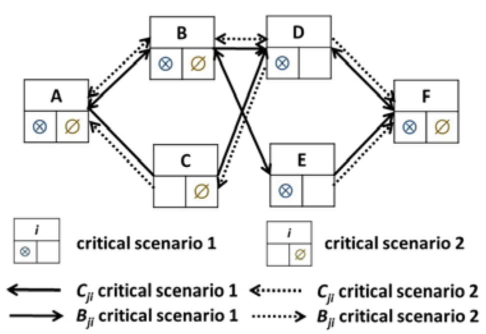
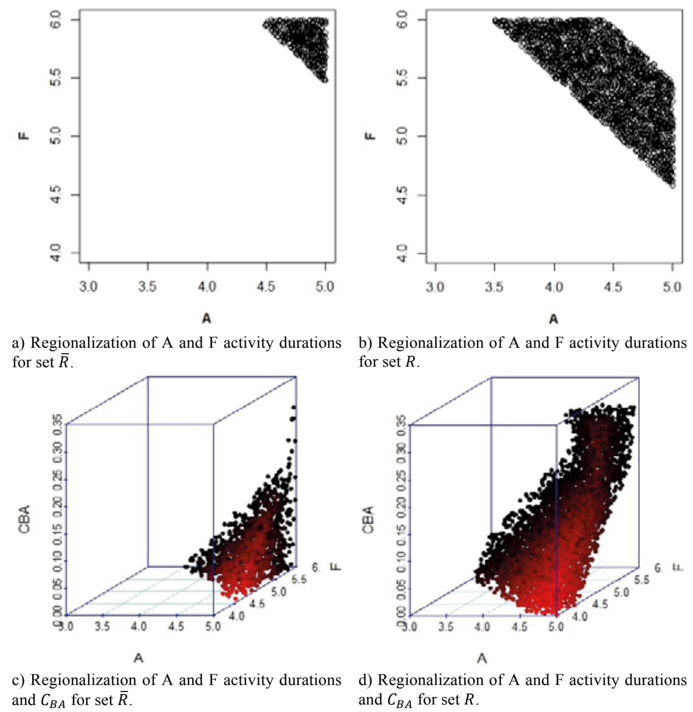


FIGURE 05. Precedence network in the case study 1.

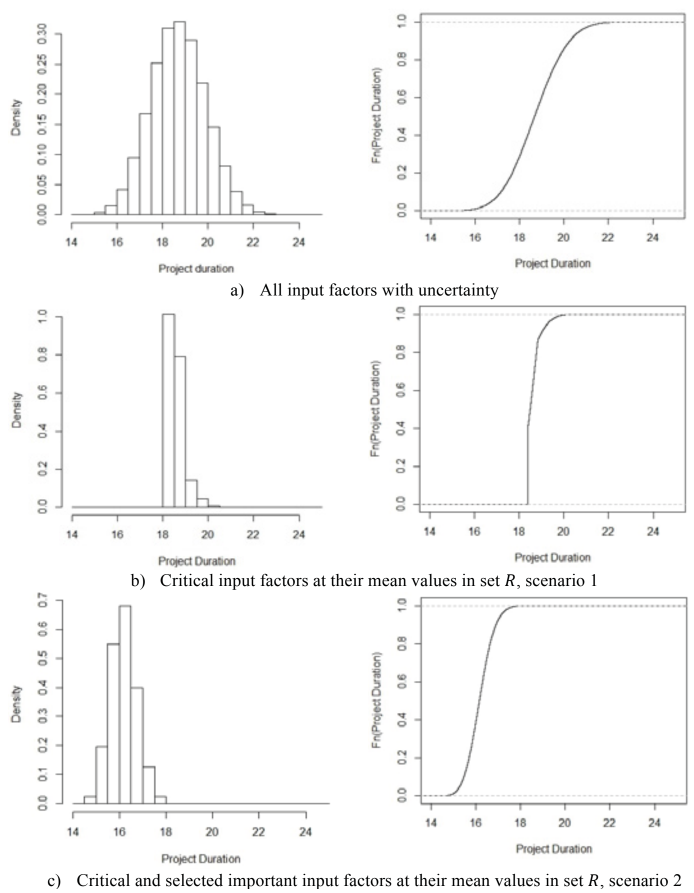
desired behavior of the project duration. However, because there are many critical variables, it is difficult to identify the regions of the sets R and $\bar{R}$. Therefore, only some variables can be regionalized. For example, in **Table 1**, the input variables with the highest values of D are the durations of activities A and F, and the overlap factor C_{BA}. Let us assume that it is desired to increase the duration of activity A (B_{AA} > 4.07) for some reason. The region of these variables that allows for the determination of the chosen project's duration can thus be studied. To this end, all variables can be fixed to the mean values of set R, and only the uncertainty of these input variables is considered. **Fig.s 6a** and **6b** show the regionalization of the A and F activity durations; it is clear that the lowest values of the A and F activity durations have better chances of yielding the desired behavior. Furthermore, it is possible to increase the duration of activity A if the duration of activity F is decreased. **Figs. 6c** and **6d** show the regionalization of the durations of activities A and F and overlap factor C_{BA}. **Fig. 6c** indicates that there are several combinations of duration values for activities A and F, and an overlap factor C_{BA} that can be used to obtain the desired behavior. For example if the duration of activities A and F are equal to 5, then an overlap factor C_{BA} over 0.20 is required. As another example, note that it is possible to increase the duration of activity A if the duration of activity F is decreased and/or the overlap factor C_{BA} is increased.

The fourth stage is the evaluation of risk of the project duration under the regionalized conditions. Let us assume that the mean values of set R is used; therefore, for the purpose of verifying that these decisions are correct, Monte Carlo simulations are performed. **Fig. 7a** shows the histogram and cumulative probability function for the project duration when uncertainty is considered in all input factors. The figure shows that project duration can take values between 14.08 and 23.80 (see also **Table 2**), and the probability that the project duration does not exceed 19.51 is 75%. **Fig. 7b** shows the results obtained when the critical variables are fixed at the mean values of set R. The probability that the project duration does not exceed 19.51 is now greater than 99% (also see **Table 2**) (or the probability of the project duration exceeding 19.51 is 1%.), which is achieved by setting the critical variables to the mean values in set R. In further analysis, Monte Carlo simulation was also performed for the opposite case. Specifically, the values of important and insignificant variables were fixed at their mean values in set R (which are almost equal to the set $\bar{R}$ because they have the same distribution), and the uncertainty in the critical variables was considered. The results show that considering uncertainty in project duration is almost the same as considering uncertainty in all variables. In conclusion, controlling the critical input variables allows for controlling the project duration between the desired values.



a) Regionalization of A and F activity durations for set $\bar{R}$.
b) Regionalization of A and F activity durations for set R.
c) Regionalization of A and F activity durations and C_{BA} for set $\bar{R}$.
d) Regionalization of A and F activity durations and C_{BA} for set R.

FIGURE 06. Regionalization of A and B activity durations and C_BA for scenario 1



a) All input factors with uncertainty
b) Critical input factors at their mean values in set R, scenario 1
c) Critical and selected important input factors at their mean values in set R, scenario 2

FIGURE 07. Monte Carlo simulation of project duration for case study 1.

	Project duration		
	No activity fixed	Scenario 1	Scenario 2
Minimum	14.08	18.39	14.39
1st Quartile	17.87	18.39	15.79
Median	18.68	18.49	16.16
Mean	18.70	18.61	16.18
3rd Quartile	19.51	18.73	16.54
Maximum	23.80	20.69	18.22

TABLE 02. Uncertainty analysis in project duration for various scenarios.

Scenario 2. Compressing the schedule for case study 1.

This scenario examines which input variables are critical for keeping the project duration between 15.0 and 16.0, thereby compressing the schedule. Considering this objective, let sets R and $\bar{R}$ be defined as follows:

R={x / 15.0≤ project duration(x) <16.0, x ∈E}

$\bar{R}$={x / 16.0≤ project duration(x) <18.0, x ∈E}

where E and project duration(x) have the same meaning as before. It is advisable not to use the full range of values greater than 16 in set $\bar{R}$ to clearly identify input variables that are responsible for behavior in restricted regions. The same procedure applied to **scenario 1** was used. The results are shown in **Table 1**, which shows that there are eight critical variables that affect the studied behavior of the project duration, seven insignificant variables that do not affect the behavior of the project duration, and five important variables that may affect the behavior of the project duration. The cumulative probability functions of all input variables are shown in Appendix B. It can be observed that the difference between the two cumulative probability functions (sets R and $\bar{R}$) for important variables B_BA, B_FE, C_CA, and C_DB is moderate, and the variables may therefore may be considered control variables. Additionally, the critical variables are not necessarily the same as those in **scenario 1**.

To obtain the desired behavior of the project duration (15≤ project duration <16), the mean values of the critical variables obtained in set R can be used. These values are as follows: B_{AA} = 3.33, B_{BB} = 5.62, B_{CC} = 4.67, B_{FF} = 4.37, B_{BA} = 0.803, B_{DB} = 0.878, B_{FD} = 0.859, B_{FE} = 0.874, C_{BA} = 0.217, C_{CA} = 0.119, C_{DB} = 0.121 and C_{DC} = 0.128. The other input variables can take any values in the uniform distributions that represent their uncertainties. **Fig. 5** shows that the critical variables change with the target value of the project duration.

As in the previous scenario, to verify that these decisions are correct, and to evaluate the risk of the project duration under the regionalized conditions, Monte Carlo simulations were performed. In **Fig. 7a**, which shows the project duration when uncertainty is considered in all input factors, the probability that the project duration does not exceed 16 is approximately 5%. **Fig. 7c** shows the results when the critical variables, and the selected important variables are fixed at their mean values from set R. The probability that the project duration does not exceed 16 is now 40%, and the probability that it does not exceed 16.54 is 75% (also see T**able 2**). In conclusion, the control of the critical input variables allows for improvement in the probability of obtaining the desired behavior of the project duration; however, there is a limit on the degree to which the probability can be improved, and the desired behaviors cannot always be achieved through the mean values of the critical variables in set R. It is necessary to perform Monte Carlo simulations to identify the likelihood of achieving the desired behavior. Additionally, the critical/important/insignificant variables depend on the desired behavior of the project duration. The identification of insignificant variables can help reduce the overall project cost because less expensive resources can be used for insignificant activities without affecting the project duration.

--- 3.2 Case 2: Maheswari and Varghese case study. ---

This case study proposed by Maheswari and Varghese (2005) considers ten activities. Maheswari and Varghese used deterministic values for all of the input parameters. Here, we use a uniform distribution function to represent the uncertainty in all input variables because we want to determine the values of activity duration and overlap factors that allow for the desired project duration to be obtained. In other words, the uniform distribution is the range of values that the input variables can take based on our manipulation. In **Table 3**, the activity duration for each activity is presented together with the dependence between activities. **Table 4** presents the uniform distribution of time factors B_{ji} and time factors C_{ji} . This problem has 40 input variables: 10 activity durations and 30 overlap time factors. For all the Monte Carlo, the SRCs and the mMCFs simulations a sample of 4,000, 400 and 12,000 points were used respectively.

The first step of the method is the simulation of project

Activity Identification	Previous Information	Duration, *U(min, max)*
A	-	*U(5.1, 6.7)*
B	*D*	*U(6.7,9.7)*
C	*A*	*U(5.6,8.1)*
D	*A, F*	*U(3.2,4.6)*
E	*B*	*U(7.7,10.0)*
F	*A, C*	*U(0.5,1.4)*
G	*F, J*	*U(1.4,2.7)*
H	*I*	*U(8.8,11.3)*
I	*D, G, E*	*U(3.9,6.0)*
J	*F, B*	*U(2.3,3.6)*

TABLE 03. List of activities with the uniform distribution of duration time (B_{ii} or A_{ii}) (Gálvez et al., 2015a).

j	*i*	Time Factors B_{ji} *U(min, max)*	Time Factors C_{ji} *U(min, max)*
C	*A*	*U(0.74,0.84)*	*U(0.07,0.16)*
F	*A*	*U(0.92,1.00)*	*U(0.00,0.12)*
F	*C*	*U(0.53,0.64)*	*U(0.26,0.36)*
D	*A*	*U(0.75,0.84)*	*U(0.05,0.14)*
D	*F*	*U(0.63,0.74)*	*U(0.15,0.25)*
B	*D*	*U(0.83,0.94)*	*U(0.31,0.44)*
J	*F*	*U(0.85,0.94)*	*U(0.14,0.25)*
J	*B*	*U(0.88,1.00)*	*U(0.00,0.10)*
G	*F*	*U(0.42,0.54)*	*U(0.42,0.54)*
G	*J*	*U(0.92,1.00)*	*U(0.26,0.36)*
E	*B*	*U(0.79,0.94)*	*U(0.07,0.17)*
I	*D*	*U(0.52,0.64)*	*U0.14,0.25)*
I	*G*	*U(0.75,0.84)*	*U(0.45,0.67)*
I	*E*	*U(0.92,1.00)*	*U(0.00,0.10)*
H	*I*	*U(0.59,1.00)*	*U(0.34,0.47)*

TABLE 04. Uniform distribution of time factors B_{ji} and time factors C_{ji}

scheduling using the DSM with equations 4 to 6 and the information in **Tables 3** and **4**. Monte Carlo simulation yields a minimum value of 27.29 for the project duration, a first quartile of 31.05, a mean of 32.08, a third quartile of 33.07, and a maximum of 37.14. Therefore, the required project duration must be between these values, given the uncertainties in the input values. The project duration follows a normal distribution even though the input variables follow a uniform distribution.

The second step of the method is the reduction in the number of input variables using GSA methods. This problem has 40 input variables; thus, it is necessary to reduce the input variables. The results of applying SRC method are presented in **Table 5**. The third column shows the absolute value of SRC for each input variable, the fourth column shows the cumulative and normalized absolute values of SRC (NS), and the fifth column shows the ranking of importance: The larger the value of absolute SRC is, the greater the effect of uncertainty of that variable on the uncertainty of the project duration becomes. In the second column, negative SRC values mean that the input variable and the project duration have different signs. Only the first 20 input variables will be considered when applying mMCF. The NS values (fourth column) indicate that these top 20 variables can explain 97% of the uncertainty in project duration.

The third and fourth steps depend on the target defined for the project duration; again, two scenarios are considered for illustration. In the first scenario, the aim is to keep the project duration between the mean and the third quartile (reduce the risk of been late). Sets R and $\bar{R}$ are defined as follows:

R={x / 32.08≤ project duration(x) <33.07, x ∈E}

$\bar{R}$={x / 33.07≤ project duration(x), x ∈E}

where E is a space formed by the uncertainties of the 20 input variables considered, whereas project duration(x) is the project duration at point x. The mMCF simulations identified 13 critical variables (B_{AA}, B_{CC}, B_{DD}, B_{BB}, B_{EE}, B_{II}, B_{HH}, B_{FC}, B_{BD}, B_{EB}, B_{HI}, C_{BD}, C_{EB}, C_{IE}, C_{HI}) that affect the desired behavior of the project duration. **Fig. 8** shows the precedence network for this case study and highlights the critical variables. Unsurprisingly, all variables are in the critical path, but the duration of activities D and F and certain overlap factors are not critical despite lying on the critical path.

Fig. 9 shows several cumulative probability functions for project duration obtained by Monte Carlo simulations. **Fig. 9a** corresponds to the case in which all 40 input variables have uncertainties. The probability that the project duration does not exceed 33.07 is 75%. **Fig. 9b** corresponds to the case in which the uncertainty is only considered in the critical variables (the other 27 input variables are fixed in their mean values). The cumulative probability functions of **Fig. 9a** and **9b** are very similar, which shows that uncertainties in critical variables are responsible for the uncertainty in project duration. **Fig. 9c** shows the Monte Carlo results obtained when the critical variables are fixed at the mean values obtained in set R and the other 27 input variables have uncertainty. The probability that the project duration does not exceed 33.07 is 80%, and the uncertainty has been reduced relative to that shown **Fig. 9a**. It is clear that controlling the critical variables can help obtain the desired project duration.

In the **second scenario**, the aim is to keep the project duration below the first quartile (compressing the schedule). Sets R and $\bar{R}$ are defined as follows:

R={x / project duration(x) <31.05, x ∈E}

$\bar{R}$={x / 31.05≤ project duration(x) <32.08, x ∈E}

In this scenario, the same procedure used in previous scenarios is applied, and 15

Input Factor	SRC	| SRC |	NS	Ranking	Input Factor	SRC	| SRC |	NS	Ranking
B_{HI}	0.388	0.388	0.10	1	C_{FC}	-0.021	0.021	0.98	21
B_{EE}	0.360	0.360	0.19	2	B_{DF}	0.021	0.021	0.98	22
B_{HH}	0.293	0.293	0.26	3	C_{GJ}	-0.010	0.010	0.99	23
B_{II}	0.289	0.289	0.34	4	B_{GF}	-0.007	0.007	0.99	24
B_{BB}	0.269	0.269	0.40	5	B_{ID}	0.006	0.006	0.99	25
C_{HI}	-0.245	0.245	0.47	6	B_{DA}	0.005	0.005	0.99	26
B_{AA}	0.238	0.238	0.53	7	C_{IG}	0.005	0.005	0.99	27
B_{EB}	0.234	0.234	0.58	8	B_{JF}	-0.005	0.005	0.99	28
B_{CC}	0.214	0.214	0.64	9	C_{FA}	-0.005	0.005	0.99	29
C_{BD}	-0.187	0.187	0.69	10	B_{JJ}	0.003	0.003	1.00	30
B_{DD}	0.177	0.177	0.73	11	C_{ID}	0.003	0.003	1.00	31
C_{EB}	-0.163	0.163	0.77	12	C_{GF}	0.003	0.003	1.00	32
B_{FC}	0.136	0.136	0.81	13	C_{DA}	0.002	0.002	1.00	33
B_{IE}	0.130	0.130	0.84	14	B_{JB}	0.002	0.002	1.00	34
C_{CA}	-0.121	0.121	0.87	15	B_{FA}	0.002	0.002	1.00	35
B_{CA}	0.108	0.108	0.90	16	B_{IG}	-0.001	0.001	1.00	36
C_{IE}	-0.093	0.093	0.92	17	C_{JF}	0.001	0.001	1.00	37
B_{BD}	0.078	0.078	0.94	18	B_{GJ}	0.001	0.001	1.00	38
C_{DF}	-0.072	0.072	0.96	19	C_{JB}	0.000	0.000	1.00	39
B_{FF}	0.060	0.060	0.97	20	B_{GG}	0.000	0.000	1.00	40

TABLE 05. Standardized regression coefficients (SRC) for case study 2.

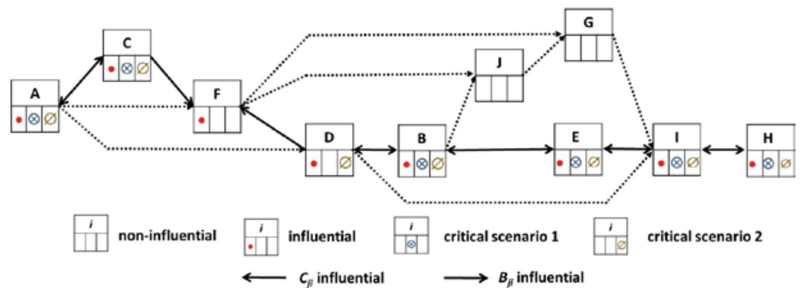
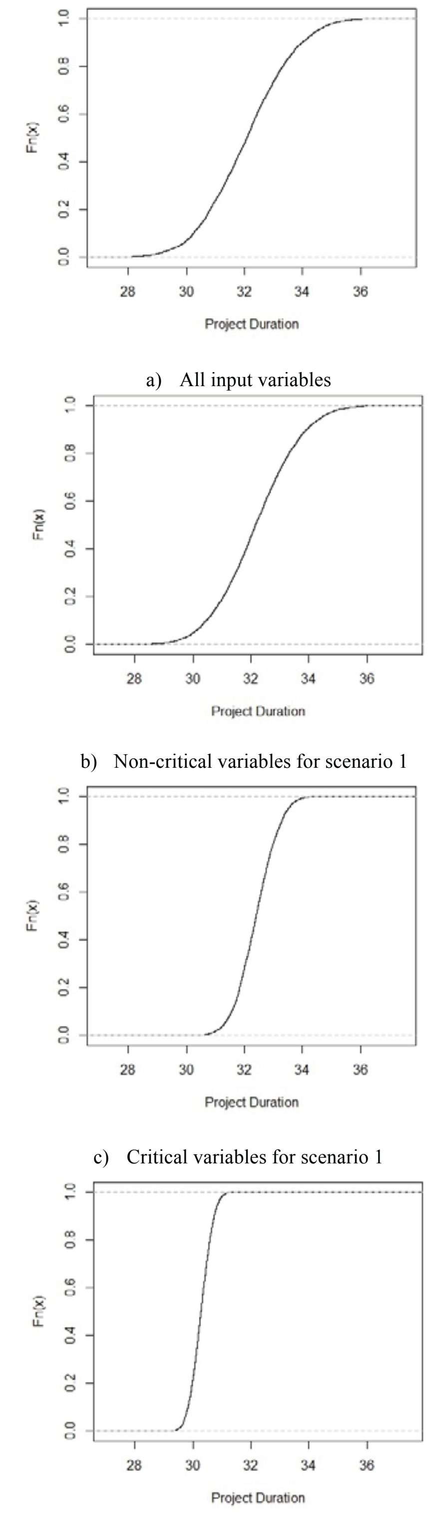


FIGURE 08. Precedence network for case study 2.

critical variables (B_{AA}, B_{CC}, B_{DD}, B_{BB}, B_{EE}, B_{II}, B_{HH}, B_{FC}, B_{BD}, B_{EB}, B_{HI}, C_{BD}, C_{EB}, C_{IE}, C_{HI}) that affect the desired behavior of the project duration are identified. **Fig. 8** shows that the durations of all activities in the critical path are critical with the exception of F activity, but only the overlaps between B and E, I and E and I and H are critical. **Fig. 9a** shows the uncertainty in all input variables, and the probability that the project duration does not exceed 31.05 is 25%. **Fig. 9d** shows the Monte Carlo results when the critical variables are fixed at the mean values obtained in set R in **scenario 2** and the other 25 input variables have uncertainty. The probability that the project duration does not exceed 31.05 is 99%. In this situation, the results are excellent, and it can be concluded that by controlling the 15 critical input variables, the desired project duration can be obtained with a probability of 99%.

--- 3.3 Road pavement project. ---

This case study considers a road pavement project (a highway of 3.53 km long,) with 26 activities, as described by previous authors (Yang, 2007; Ioannou and Martinez, 1998; Brand et al., 1964). The construction work for this highway is distributed in two sections (one from station 42 to station 100 and another from station 100 to station 158) based on the position of the balance points for cut and fill (earthmov-



a) All input variables

b) Non-critical variables for scenario 1

c) Critical variables for scenario 1

d) Critical variables for scenario 2

FIGURE 09. Cumulative distribution functions for several scenarios in case study 2. a) All input variables with uncertainty; b) non-critical variables for scenario 1 fixed at their mean values; c) critical variables for scenario 1 fixed at their mean values in set R; d) critical variables for scenario 2 fixed at their mean values in set R.

ing). In each section, the quantity of cut matches the quantity of fill. **Table 6** shows the names of the activities, the distribution functions of activity durations, and direct predecessors. **Fig. 10** shows the precedence network for the road pavement. Ioannou and Martinez (1998) used traditional three-point estimates (optimistic, most-likely, pessimistic) for activity durations; they assumed that the necessary resources are available and, therefore, that there is no correlation between activities. Yang (2007) used triangular (minimum, most likely, maximum), uniform and discrete distribution functions for the duration of activities and assumed that there are correlations between activities because some activities are performed by the same crew. However, these studies were related to uncertainty analysis. The objective here is different: Given a desired project duration behavior, the critical variables and their values or regions must be identified. Therefore, the distribution functions for activity duration (**Table 6**) represent the potential duration of these activities when different resources (crew, equipment and technologies) are used. Uniform distribution function indicates that the activity duration can take any value between the minimum and maximum values. Discrete distribution function indicates that only some values

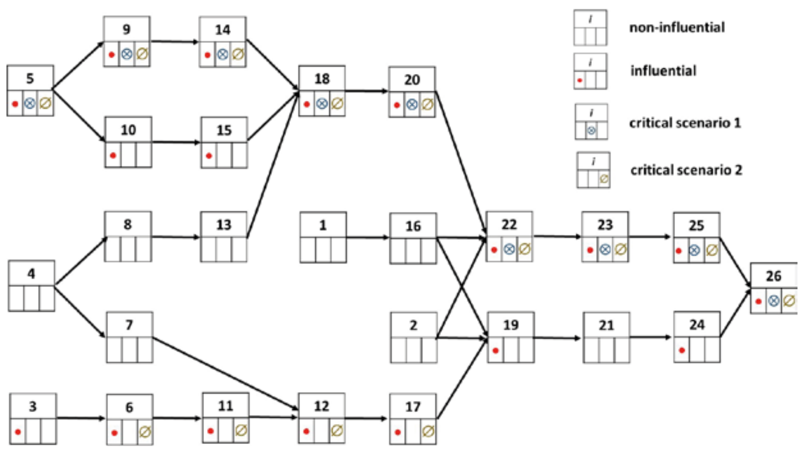


FIGURE 10. Activity precedence network in the highway construction project.

ID	Activity	Duration	Predecessor
1	Setup batch plant	U (0.5, 3.5)	-
2	Order & deliver paving mesh	U(2, 8)	-
3	Deliver rebar for double barrel culvert	U(2.5, 11.5)	-
4	Move in equipment	U(1.5, 4.5)	-
5	Deliver rebar for small box culvert	U(1, 25)	-
6	Build double barrel culvert	D(4, 8, 10, 16)	3
7	Clear & grub from Sta. 42 to Sta. 100	D(1, 2.5, 7)	4
8	Clear & grub from Sta. 100 to Sta. 158	D(2.5, 7, 11.5)	4
9	Build box culvert at Sta. 127	U(1, 13)	5
10	Build box culvert at Sta. 138	U(0.5, 9.5)	5
11	Cure double barrel culvert	U(3, 15)	6
12	Move dirt between Sta. 42 & Sta. 100	U(2.5, 11.5)	7,11
13	Start moving dirt between Sta. 100 & Sta. 150	U(1.5, 4.5)	8
14	Cure box culvert at Sta. 127	U(1.5, 28.5)	9
15	Cure box culvert at Sta. 138	U(2, 14)	10
16	Order & stockpile paving material	U(0.5, 3.5)	1
17	Place subbase from Sta. 42 to Sta.100	U(3.6, 14)	12
18	Finish moving dirt between Sta. 100 & Sta. 150	U(1, 13)	13, 14, 15
19	Pave from Sta. 42 to Sta. 100	U(4, 16)	2, 16, 17
20	Place subbase from Sta. 100 to Sta. 158	U(2, 21.87)	18
21	Cure pavement from Sta. 42 to Sta. 100	U(2.5, 11.5)	19
22	Pave from Sta. 100 to Sta. 158	U(3, 30)	2, 16, 20
23	Cure pavement from Sta. 100 to Sta. 158	U(2.5, 11.5)	22
24	Place shoulders from Sta. 42 to Sta. 100	U(1,7)	21
25	Place shoulders from Sta. 100 to Sta. 158	U(1,7)	23
26	Place guardrail & landscape	D(2.5, 4, 11)	24, 25

TABLE 06. Activity list, duration distribution functions in days, and predecessor for road pavement project. (U represent uniform distribution; D discrete distribution)

are possible. Here, all discrete values have the same probability; therefore, the duration of activity 6 can take 4, 8, 10, or 16 days, each with a probability of 25%. Additionally, it is assumed that the necessary resources are available, i.e., that there is no correlation between activities. For all the Monte Carlo simulations, all the SRCs, and all the mMCFs a sample of 10,000, 2,500 and 12,500 points were used respectively.

First, project scheduling is simulated using the DSM using equations 1 to 3 because in this case there is not overlap between activities. Monte Carlo simulation gives a minimum value of 44.30, a first quartile of 79.77, a mean of 89.94, a third quartile of 99.41, and a maximum of 143.75 days for project duration. This information yields the number of days and the probability of a given project duration based on the potential values of activity durations.

The second step of the method is the reduction in the number of input variables using the SRC method for global sensitivity analysis. The results obtained by applying SRC are presented in **Table 7**. The third column shows the absolute value of SRC for each input variable, the fourth column shows the cumulative and normalized absolute values of SRC (NS), and the fifth column shows the ranking of importance: The larger the value of absolute SRC is, the greater the effect of uncertainty of that variable on the uncertainty of the project duration becomes. Only the first 18 input variables will be considered when applying mMCF because these 18 input variables explain over 97% of the uncertainties in the project duration (see NS values). **Fig. 10** also highlights the influential/non-influential activity durations.

The third and fourth steps depend on the aim defined for the project duration; again, two scenarios are considered for illustration: **scenario 1** reducing the risk of late delivery for the road pavement project, and **scenario 2** compressing the schedule.

The third step in the method is the determination and regionalization of critical variables using the modified Monte Carlo filtering. Eighteen influential variables are considered in this study, which have the strongest effect on whether the project duration exceeds 99.41 days (third quartile) for **scenario 1**, or remains under 84 days for **scenario 2**. The sets R and $\bar{R}$ are thus defined as follows:

$R=\{x \,/\, 89.94 \leq \text{project duration}(x) < 99.41,\ x \in E\}$

$\bar{R}=\{x \,/\, 99.41 \leq \text{project duration}(x),\ x \in E\}$

For scenario 1, and

$R=\{x \,/\, \text{project duration}(x) < 84,\ x \in E\}$

$\bar{R}=\{x \,/\, 84 \leq \text{project duration}(x) < 95,\ x \in E\}$

For scenario 2.

The mMCF is applied by taking the uncertainty ranges given in **Table 6** for the 18 influential input variables, and the

Input Variable	SRC	| SRC |	SN	Ranking	Input Variable	SRC	| SRC |	SN	Ranking
22	0.5013	0.5013	0.1711	1	11	0.0296	0.0296	0.9541	14
5	0.4407	0.4407	0.3215	2	17	0.0283	0.0283	0.9637	15
14	0.4235	0.4235	0.4660	3	24	0.0239	0.0239	0.9719	16
20	0.3622	0.3622	0.5896	4	12	0.0232	0.0232	0.9798	17
26	0.2287	0.2287	0.6677	5	3	0.0181	0.0181	0.9860	18
18	0.2186	0.2186	0.7423	6	21	0.0180	0.0180	0.9921	19
9	0.1778	0.1778	0.8029	7	16	-0.0082	0.0082	0.9949	20
23	0.1549	0.1549	0.8558	8	8	-0.0045	0.0045	0.9964	21
25	0.1072	0.1072	0.8924	9	1	-0.0044	0.0044	0.9979	22
6	0.0496	0.0496	0.9093	10	2	-0.0024	0.0024	0.9987	23
15	0.0399	0.0399	0.9229	11	4	-0.0022	0.0022	0.9995	24
19	0.0312	0.0312	0.9335	12	7	-0.0010	0.0010	0.9998	25
10	0.0305	0.0305	0.9440	13	13	0.0004	0.0004	1.0000	26

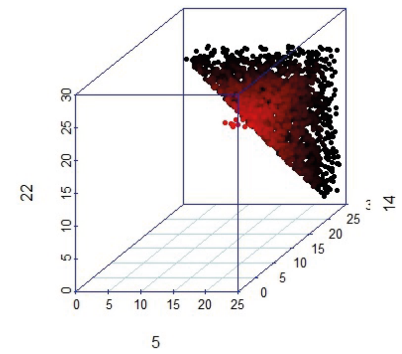
TABLE 07. Standardized regression coefficients (SRCs) for the road pavement project

values of the non-influential activities are fixed in the nominal values. The density functions associated with each variable are compared using the Kolmogorov-Smirnov test. The results are displayed in **Table 8**, which shows that there are nine critical variables for scenario 1 (and 13 for scenario 2) that affect the desired behavior of the project duration, eight (four) insignificant variables that do not affect the desired behavior of the project duration, and one (one) important variable that may affect the desired behavior of the project duration. Here, the important variable is not considered because the cumulative probability functions of the sets R and $\bar{R}$ are very similar (see Appendix C). **Fig. 10** also highlights the critical activities for scenario 1 and 2.

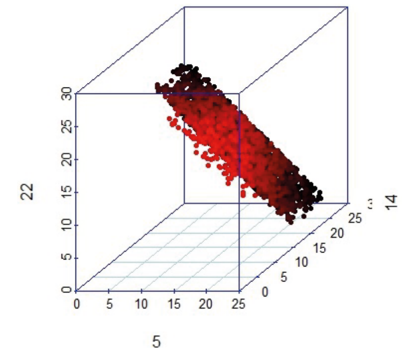
The mean values of the critical variables obtained in set R can be used to obtain the desired behavior of the project duration. These values, in **scenario 1**, for activities 5, 9, 14, 18, 20, 22, 23, 25, and 26 are 14.441, 7.256, 16.815, 7.369, 13.084, 19.048, 7.143, 4.171 and 8.0, respectively. For **scenario 2**, these values for activities 5, 6, 9, 11, 12, 14, 17, 18, 20, 22, 23, 25 and 26 are 9.910, 6, 6.386, 8.855, 6.905, 11.392, 8.682, 6.175, 9.863, 12.202, 6.544, 3.817, and 5, respectively. Additionally, the regionalization of the critical variables can be analyzed to obtain the desired behavior of the project duration. For example for **scenario 1**, the durations of activities 22, 5 and 14 can explain 46.6% of the project duration variability. The region of these variables that allows for the chosen project duration to be obtained

Scenario 1				Scenario 2			
Input Variable	D	p-values	Input variable is	Input Variable	D	p-values	Input variable is
3	0.0318	0.1144	insignificant	3	0.0252	0.1710	insignificant
5	0.1820	0.0000	critical	5	0.2254	0.0000	critical
6	0.0080	1.0000	insignificant	6	0.0433	0.0014	critical
9	0.1021	0.0000	critical	9	0.0645	0.0000	critical
10	0.0214	0.5352	insignificant	10	0.0170	0.6291	insignificant
11	0.0151	0.9016	insignificant	11	0.0473	0.0003	critical
12	0.0351	0.0617	important	12	0.0394	0.0049	critical
14	0.2154	0.0000	critical	14	0.2052	0.0000	critical
15	0.0172	0.7975	insignificant	15	0.0206	0.3829	insignificant
17	0.0221	0.4915	insignificant	17	0.0426	0.0017	critical
18	0.1182	0.0000	critical	18	0.1110	0.0000	critical
19	0.0238	0.3969	insignificant	19	0.0350	0.0172	important
20	0.1706	0.0000	critical	20	0.1521	0.0000	critical
22	0.2209	0.0000	critical	22	0.2542	0.0000	critical
23	0.0711	0.0000	critical	23	0.0818	0.0000	critical
24	0.0128	0.9745	insignificant	24	0.0160	0.7036	insignificant
25	0.0508	0.0013	critical	25	0.0652	0.0000	critical
26	0.0751	0.0000	critical	26	0.1165	0.0000	critical

TABLE 08. Modified Monte Carlo filtering results for road pavement project.



a) Regionalization of duration of activities 22, 5 and 14 for set $\bar{R}$.



b) Regionalization duration of activities 22, 5 and 14 for set R.

FIGURE 11. Regionalization of activities 22, 5 and 14 for scenario 1 in road pavement project.

can therefore be studied. To this end, all variables can be fixed in the mean values of set R, and only the uncertainty of these input variables is considered. **Figs. 11a** and **11b** show the regionalization of the durations of activities 22, 5 and 14. It is clear that several combinations of the activity durations can be used to obtain the desired behavior.

The fourth stage is the evaluation of the risk of the project duration under the regionalized conditions. For this purpose, Monte Carlo simulations were performed using the mean values of set R for the critical variables; full uncertainties were used for the non-critical variables (insignificant, important and non-influential variables). **Fig. 12a** shows the cumulative probability function for the project duration when uncertainty is considered in all input factors. The figure shows that the probability of the project duration not exceeding 99.41 and 84 days is 75% and 25% respectively. **Fig. 12b** shows the results when the critical variables are fixed at the mean values of set R for scenario 1. The probability that the project duration does not exceed 99.41 is 100%. In fact, the project duration takes the deterministic value of 97.3 days—an excellent result. **Fig. 12c** shows the results when the critical variables are fixed at the mean values of set R for **scenario 2**. The probability that the project duration does not exceed 84 days is 75%—a good result.

4. DISCUSSION

The methodology developed can be used to answer related questions: Which variables are responsible for lateness? What activities should be accelerated to compress a schedule? What activities should be overlapped to compress a schedule? If the project has hundreds or thousands of activities, the mMCF (**stage 3**) may not be applied unless in **stage 2** the number of input variables is reduced to around twenty. If there are many input variables, then it is necessary to identify milestones and divide the project into smaller projects or sub-projects. If there are interdependencies between these sub-projects, the method can be applied to the project as represented by sub-projects and to each sub-project separately. However, the effect of the number of input variables in mMCF should be studied; indeed, this will be the aim for future studies on this topic.

The results of applying mMCF depend on the sample size. In the examples discussed, several sample sizes were considered, and for each sample size, several samples were used. This program was executed several times. The sample size that provided the best reproducibility was selected. In general, the critical and insignificant variables tend to be the same between different samples, but the important variables often change to critical or insignificant types in each sample, based on the Kolmogorov-Smirnov test. Thus, it is advisable to observe the cumulative probability functions, which provide more reproducible results. In other words, the results obtained with the Kolmogorov-Smirnov test must be confirmed by a visual review of the cumulative probability functions.

Based on the case studies, it can be stated that controlling critical variables tends to yield the desired results with a high level of probability, but it is necessary to verify the results by using the Monte Carlo simulation to determine the probability of obtaining the desired behavior. Furthermore, it is clear that in practice, not all input variables may be controlled, or the controls can have high costs and may therefore be infeasible in practice. In this case, regionalization between the variable in question and the variables that most strongly affect the project duration can be achieved; two or three variables typically represent a good size for visual study. This regionalization can be achieved with several pairs and/or trios of variables.

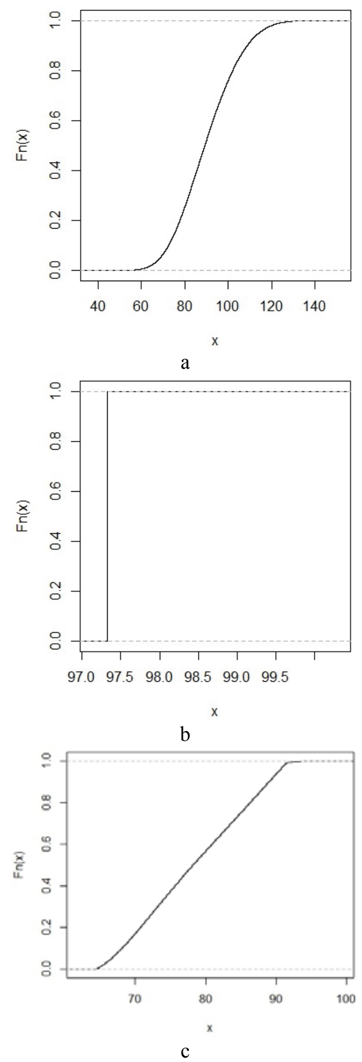


FIGURE 12. Cumulative distribution functions for project duration (x) for road pavement project- a) all input variables with uncertainty; b) critical variables for scenario 1 fixed at their mean values; b) critical variables for scenario 2 fixed at their mean values;

The method can be extended to more than one output variable, e.g., project duration and project cost. In such cases, a model that includes all output variables is required. Global sensitivity methods are readily applicable to more than one output variable and can therefore be applied to reduce the number of input variables. However, the application of mMCF to more than one output variable is not straightforward, and further study in this respect is required.

5. CONCLUSIONS

A method was presented to identify and localize critical variables for project scheduling under uncertainty. The proposed method consists of four stages: 1) a Monte Carlo simulation of project scheduling using a DSM, 2) the reduction in the number of input variables using an index based on the standardized regression coefficient, 3) the determination and regionalization of critical variables with a modified MCF method, and 4) an evaluation of risk of the project duration under the regionalized conditions. In addition, several contributions were included: 1) the method of Maheswari and Varghese was extended to include the calculation of late start, late finish, and slack time and examples of calculation were included; 2) the cumulative standardized regression coefficient was introduced as an index to reduce the number of input variables; 3) the MCF method was modified so that the size of the sets of desired behavior and unwanted behavior have similar sizes, which improves the statistical behavior of the method; and 4) criteria was developed for the regionalization of the input variables.

The method was implemented in three case studies including projects with and without overlapping, uniform and discrete distribution functions to represent uncertainties in input variables, activity acceleration and/or overlapping as strategies for compressing the schedule. Based on these case studies, it was demonstrated that the methodology proposed allows for the identification of the input variables responsible for a certain behavior of the project duration. In addition it was demonstrated that: 1) the Monte Carlo simulation of project duration using DSM allows for the identification of the feasible behavior of the project duration and the identification of the adequate definition of desired behavior and unwanted behavior for the MCF method; 2) the cumulative SRC can be used to identify the non-influential input variables for the uncertainty of the project duration. Then, these input variables can be fixed in the nominal values. This means that the model to estimate the project duration will have less input variables, which improves the result given by the mMCF; 3) the modified MCF method has proven to be a good method for the identification of critical variables to obtain the desired duration of a project; 4) by using the mean values of the critical variables in the desired behavior set, the desired behavior is obtained with a high probability. ◆

• APPENDIX •

APPENDIX A

--- Examples of project programming using the DSM ---

This appendix presents two examples of activity programming using the DSM. Both examples consider a project with six activities (A,B,C,D and F): The first involves a programming problem without overlap, and the second involves a programming problem with overlap.

Example without overlap. The DSM for this example is shown in **Fig. A1**. Values in the diagonal are the activity duration, and "x" represents the dependence between activities. **Fig. A1** also includes three additional columns representing the early start (ES), late start (LS) and total slack time or total float time (S) for each activity. Moreover, two rows showing the early finish (EF) and late finish (LF) for each activity are included. The required equations are

$(EF)_i=(ES)_i+A_{ii}$	$0<i\leq n$	(A1)
$(ES)_j=\text{Max}\,[(EF)_i]$	$0<i\leq n, 0<j\leq n$	(A2)
$P=\text{Max}\,[(EF)_i]$	$0<i\leq n$	(A3)
$(LS)_i=(LF)_i-A_{ii}$	$0<i\leq n$	(A4)
$(LF)_i=\text{Min}\,[(LS)_j]$	$0<i\leq n, 0<j\leq n$	(A5)
$S_i=(LS)_i-(ES)_i$	$0<i\leq n$	(A6)

We start with the activities that do not have predecessors, $(ES)_A=0$, and using equation A1 $(EF)_A$ is calculated by adding the activity duration ($A_{AA}=2$) to the $(ES)_A$. Therefore, $(EF)_A$ is equal to 2, and $(ES)_B$ is equal to $(EF)_A$ because the activity A is predecessor of activity B. **Fig. A1** shows the calculation of $(EF)_B$ and $(ES)_D$, which can be easily done by hand or using a spreadsheet.

After the early start and early finish have been calculated for all activities, the calculation of late start and late finish can be considered using equations A4 and A5. The calculation begins with the activities without successor activities, in this case activity F. Therefore, $(LF)_F=14$, and the late start of activity F is equal to the late finish minus the F activity duration. **Fig. A2** illustrates the calculation of $(LF)_B$ and $(LF)_F$. It can be observed that the calculation of early/late finish/start can be easily done in the DSM by hand or using a spreadsheet. Finally, the total slack time can be calculated using equation A6, as shown in **Fig. A2**.

Fig. A2 is adequate for observing all the information in a matrix: activity duration, dependency, ES, LS, EF, LF and S.

Example with overlap. This example considers overlap between activities. **Fig. A3** shows a DSM that includes B_{ji}, C_{ji} overlap factor times and the activities durations (B_{ii} or C_{ii}). The equations required for the calculation of early/late start/finish are as follows:

$(ES)_j=\text{Max}\,[(ES)_i+B_{ji}B_{ii}-C_{ji}C_{jj}]$	$0<i\leq n, 0<j\leq n$	(A7)
$(EF)_i=(ES)_i+B_{ii}$	$0<i\leq n$	(A8)
natural overlap project duration $=\text{Max}[(EF)_i]$	$0<i\leq n$	(A9)
$(LS)_i=\text{Min}[(LS)_j-B_{ji}B_{ii}+C_{ji}C_{jj}]$	$0<i\leq n, 0<j\leq n$	(A10)
$(LF)_j=(LS)_j+B_{jj}$	$0<i\leq n$	(A11)
$S_j=(LF)_j+(LS)_j$	$0<i\leq n$	(A12)

Again, the calculation begins by the early start of activities without a predecessor, $(ES)_A=0$,

$(EF)_B=(ES)_B+A_{BB}=2+4=6$

	A	B	C	D	E	F	ES	LS	S
A	2						0		
B	x	4					2		
C	x		3.5				2		
D		x	X	5			6		
E		x			4		6		
F				x	x	3	11		
EF	2	6	5.5	11	10	14			
LF									

$(ES)_D=Max[(EF)_B,(ES)_C]=Max[6,5.5]=6$

FIGURE A1. Early start and early finish calculation for the example without overlap

$(LF)_B=Min[(LS)_E,(LS)_D]=Min[7,6]=6$

	A	B	C	D	E	F	ES	LS	S
A	2						0	0	0
B	x	4					2	2	0
C	x		3.5				2	2.5	0.5
D		x	x	5			6	6	0
E		x			4		6	7	1
F				x	x	3	11	11	0
EF	2	6	5.5	11	10	14			
LF	2	6	6	11	11	14			

$(LS)_E=(LF)_E-A_{EE}=11-4=7$

FIGURE A2. Late start and late finish calculation for example without overlap.

$(ES)_B=Max[(ES)_A+B_{BA}\,B_{AA}-C_{BA}\,C_{BB}]=Max[0+0.87\times2-0.13\times4]=1.22$

	A	B	C	D	E	F	ES	LS	S
A	2						0	0	0
B	0.87 / 0.13	4					1.22	1.25	0.03
C	0.95 / 0.05		3.5				1.73	1.73	0
D		0.95 / 0.05	0.95 / 0.05	5			4.8	4.8	0
E		0.95 / 0.05			4		4.82	5.75	0.93
F				0.95 / 0.05	0.95 / 0.05	3	9.4	9.4	0
EF	2	5.22	5.23	9.8	8.82	12.4			
LF	2	5.25	5.23	9.8	9.75	12.4			

$(EF)_D=(ES)_D+B_{DD}=4.8+5=9.8$

FIGURE A3. ES, LS, EF, LF and S for example with overlap

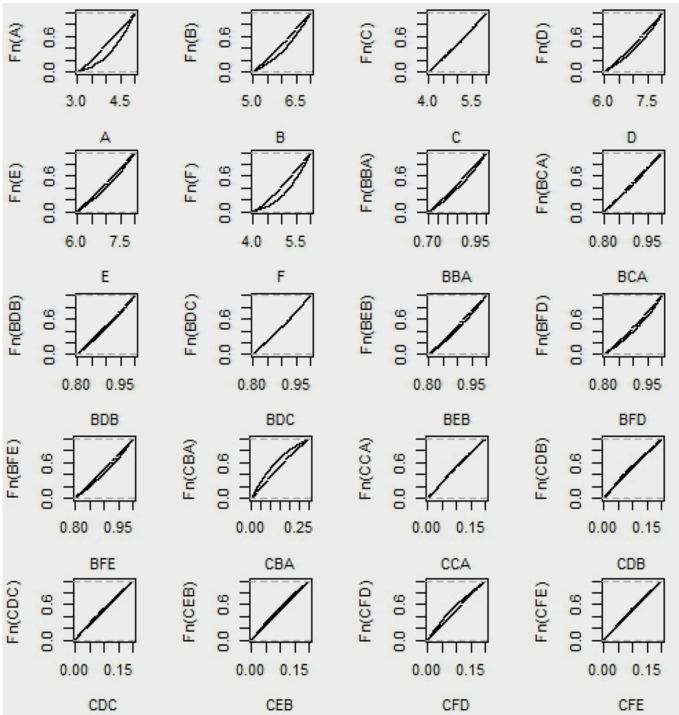
and the early finish is easily calculated as follows: $(EF)_A=(ES_A+B_{AA}=2$. These values are thus included in the corresponding column and row. This is done for all activities. **Fig. A3** explains how to make these calculations by hand or using a spreadsheet. The project duration is therefore the maximum value of early finish, in this case 12.4. The late start and late finish can be calculated by starting with the activities without a successor, in this case activity F. The calculations of late start and late finish are not show, but the readers can refer to equations A10 and A11. Note that **Fig. A3** is a matrix representation including both overlap factors that allows for the calculation of ES, LS, EF, LF and S. Additionally, this matrix allows for all the information to be viewed simultaneously.

• APPENDIX •

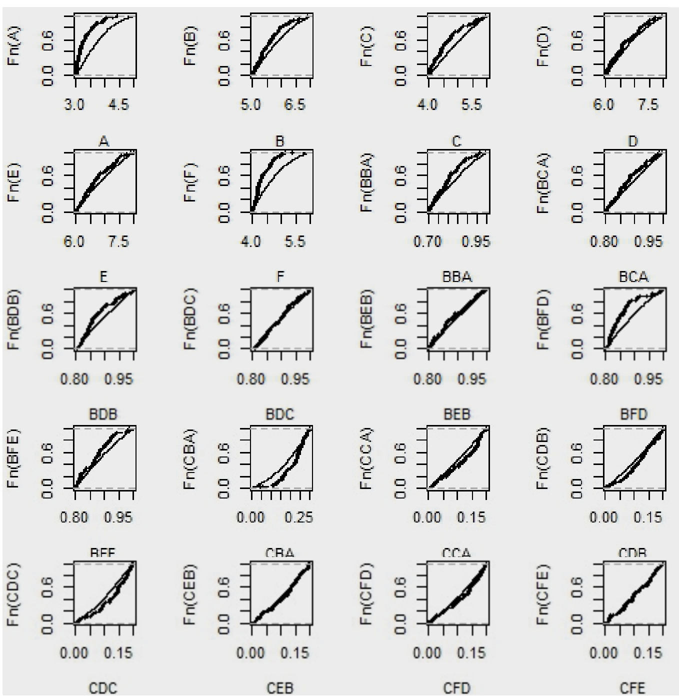
APPENDIX B

--- Cumulative distribution functions for sets R and $\bar{R}$ in case study 1. ---

Scenario 1



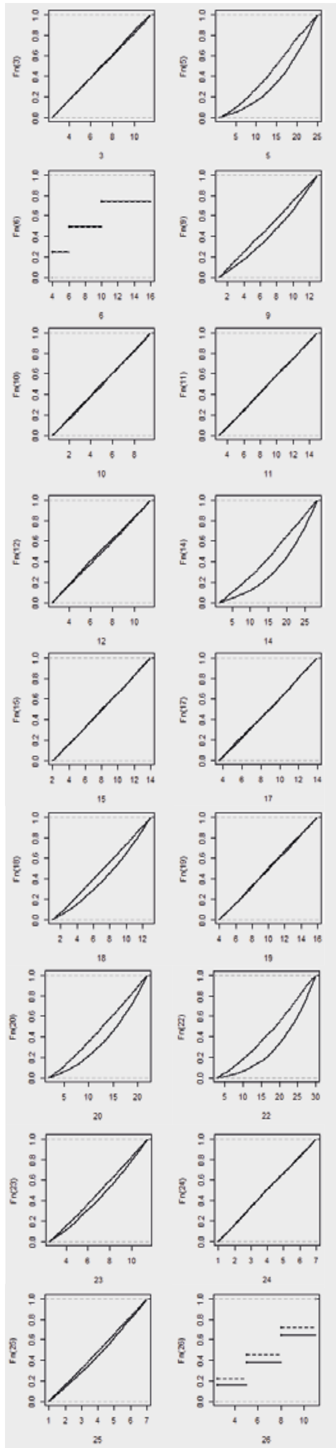
Scenario 2



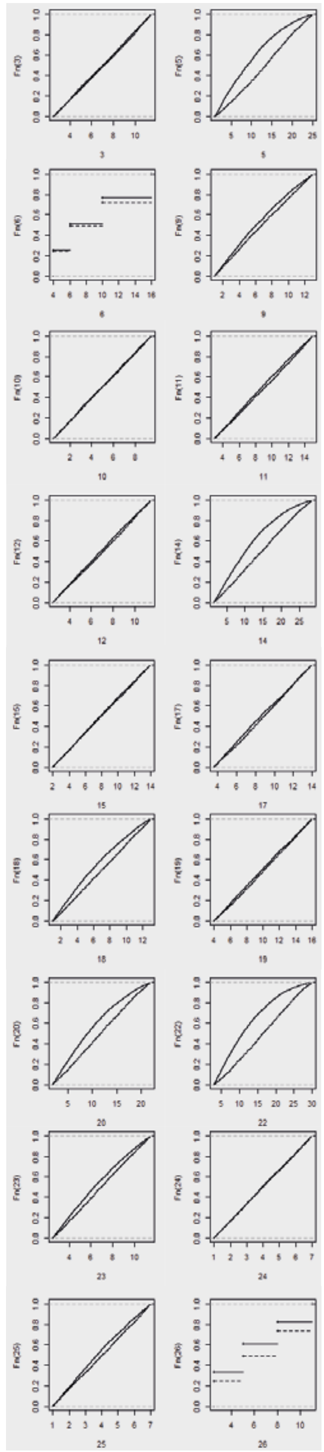
APPENDIX C

--- Cumulative distribution functions for sets R and $\bar{R}$ in road pavement project ---

Scenario 1



Scenario 2



• AUTHORS •

EDELMIRA D. GÁLVEZ PhD in Project Engineering at Universitat Politècnica de València (2015, Spain) and PD degree at the University of Wisconsin-Madison (1994, USA). She graduated in Metallurgical Engineering (Universidad Católica del Norte, 1991) and Industrial Engineering (Universidad de Antofagasta,1999). Professor Gálvez joined the Department of Mines and Metallurgical Engineering, Universidad Católica del Norte (Chile), in 1999 as assistant professor. Professor Gálvez´s principal research interest is the use of a system approach to solving problems in mineral process, design and analysis. Professor Gálvez has published 34 peer reviewed journal articles, more than 30 conference papers, and more than 15 book chapters.

SALVADOR F. CAPUZ-RIZO, PhD in Industrial Engineering and Professor of Project Management and Engineering since 2003 at Universitat Politècnica de València. He graduated in Industrial Engineering, major in Electricity, in January 1990. Later on, in April 1991, he obtained the Master's Degree in Computer Assisted Design and Integrated Manufacturing and in 1994, completed his PhD in Industrial Engineering (Universitat Politècnica de València). Professor Capuz-Rizo has published 20 peer reviewed journal articles, more than 140 conference papers, and is author or co-author of 9 books. His main research interest are Project Management and Environmental Product Design & Eco-efficiency. Currently serves as President of AEIPRO (Spanish Project Management and Engineering Association), Spanish Member of IPMA (International Project Management Association).

JOAQUIN B. ORDIERES-MERE is IPMA-B, PMP,PhD in industrial Engineering and Full Professor of Project Management since 1997. He worked for several universities and currently he works at the Technical University of Madrid. He was graduated by 1985 in Industrial Engineering, major in Mechanics (University of Oviedo) and he completed his PhD from the National University for Distant Studies in 1988. Professor Ordieres has published more than 100 peer reviewed papers, more than 150 conference papers, he is co/author of 15 books, and he also holds the co-authorship in more than ten patents. He serves a a regular evaluator for the EU call of proposals and he belongs to the EU TGS8 expert committee for monitoring projects but also for several journals and he belongs to the scientific committee of the IJDMMM journal. Professor Ordieres' main research interests are connect to Project Knowledge and Project Management but, in a broader sense, to the societal impact of management, specially when foresee from data. In connection to this, all the relevant topics related to Big-Analytics in industry looking to doing better with energy awareness as well as in pollution are under his interest. Regarding normalization activities, he is an active member serving to the Spanish normalization body (AENOR), specially regarding the activities of the ISO TC258, dealing with project management standardization activities.

• REFERENCES •

Browning, T.R. (2001), Applying the design structure matrix to system decomposition and integration problems: A review and new directions. IEEE Transactions on Engineering management, 48 (3) , 292-306.

Brand, J.D., Meyer, W.L., & Shaffer, L.R.. (1964). The resource scheduling problem in construction. Civil Engineering Studies, Construction Research Series No. 5. University of Illinois, Urbana, Illinois.

Brockmann, D., & Morgenroth, E. (2010). Evaluating operating conditions for outcompeting nitrite oxidizers and maintaining partial nitrification in biofilm systems using biofilm modeling and Monte Carlo filtering, Water Research, 44, 1995-2009.

Carr, C.R. (1979). Simulation of construction project duration, Journal of Construction Division, ASCE, 105, 117-128.

Gálvez, E.D., Capuz-Rizo, S.F., & Ordieres, J.B., (2012). Study of the uncertainty of task programming using the dependency structure matrix, Información Tecnológica, 23(1), 19-34.

Gálvez, E.D., Ordieres, J.B., & Capuz-Rizo, S.F., (2015a). Evaluation of Project Duration Uncertainty using the Dependency Structure Matrix and Monte Carlo Simulations, Revista de la Construcción, 14(2), 72-79.

Gálvez, E.D., Ordieres, J.B., & Capuz-Rizo, S.F., (2015b). Analysis of Project Duration Uncertainty using Global Sensitivity Analysis, Journal of Modern Project Management, 2(3), 18-25.

Gálvez, E.D., & Capuz-Rizo, S.F., (2016). Assessment of global sensitivity analysis methods for project scheduling, Computers and Industrial Engineering, 93, 110-120.

Gálvez, E.D., Ordieres, J.B., & Capuz-Rizo, S.F., (2017). On uncertainty and sensitivity analyses in project duration based on dependency information, Journal of Modern Project Management, 4(3), 98-109.

Gonzalez-Nicolas, A., Bau, D., Cody, B. M., & Alzraiee, A., (2015). Stochastic and global sensitivity analyses of uncertain parameters affecting the safety of geological carbon storage in saline aquifers of the Michigan Basin, International Journal of Greenhouse Gas Control, 37, 99-114.

Herroelen, W., & Leus, R., (2005). Project scheduling under uncertainty: survey and research potentials, European J. of operational research, 165, 289-306.

Ioannou, P.G., & Martinez, J.C., (1998). Project scheduling using state-based probabilistic decision networks, in D.J. Medeiros, E.F. Watson, J.S. Carson, M.S. Manivannan (eds.), Proceedings of the 1998 Winter Simulation Conference, 1287-1294.

Jarrett, A. M., Liu, Y., Cogan, N. G., & Hussaini, M. Y., (2015). Global sensitivity analysis used to interpret biological experimental results, Journal of Mathematical Biology, 71(1), 151-170.

Lim, T.C., Yi, D., Lee, D., & Arditi, D., (2014). Concurrent construction scheduling simulation algorithm, Computer-Aided Civil and Infrastructure Engineering, 29, 449-463.

Lucay, F., Cisternas, L.A., & Gálvez, E.D., (2015). Global sensitivity analysis for identifying critical process design decisions, Chemical Engineering Research and Design, 103, 74-83.

Maheswari, J.U., & Varghese, K., (2005). Project scheduling using dependency structure matrix, International Journal of Project Management, 23 (3), 223-230.

Morris M.D. (1991). Factorial Sampling Plans for Preliminary Computational Experiments, Technometrics, 33(2), 161-174.

Pujol, G., Iooss, B., & Janon, A., (2014). Sensitivity: Sensitivity Analysis. R package version 1.10.1., <http://CRAN.R-project.org/package=sensitivity>.

R Core Team. (2013). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria, <http://www.R-project.org/>.

Saltelli, A., Tarantola, S., Campolongo, F., & Ratto, M., (2004). Sensitivity analysisin practice. A guide to assessing scientific models. John Wiley & Sons.

Sathyanarayanamurthy H., & Chinnam, R.B., (2009). Metamodels for variable importance decomposition with applications to probabilistic engineering design, Computers & Industrial Engineering, 57, 996–1007.

Sepúlveda, F.D., Cisternas, L.A., & Gálvez, E.D., (2014). The Use of Global Sensitivity Analysis for Improving Processes: Applications to Mineral Processing, Computers & Chemical Engineering, 66, 221-232

Shannon, R. J., Tomlin, A. S., Robertson, S. H., Blitz, M. A., Pilling, M. J., & Seakins, P. W., (2015). Global Uncertainty Propagation and Sensitivity Analysis in the CH3OCH2 + O-2 System: Combining Experiment and Theory To Constrain Key Rate Coefficients in DME Combustion, Journal of Physical Chemistry A, 119(28), 7430-7438.

Shi, Q., & Blomquist, T., (2012). A new approach for project scheduling using fuzzy dependency structure matrix, International Journal of Project Management, 30, 503-510.

Sobol' I.M., (1993). Sensitivity analysis for non-linear mathematical model, Math. Modelling Comput. Exp. 1, 407–414.

Spear, R.C., & Hornberger, G.M., (1980). Eutrophication in peel inlet. 2. Identification of critical uncertainties via generalize sensitivity analysis. Water Research, 14 (1), 43–49.

Spear, R.C., Grieb, T.M., & Shang, N., (1994). Factor uncertainty and interaction in complex environmental models, Water Resour. Res., 30, 3159-3169.

Srour, I.M., Abdul-Malak, M.U., Yessine, A.A., & Ramadan, M., (2013). A methodology for scheduling overlapped design activities based on dependency information, Automation in Construction, 29, 1-11.

Vanrolleghem, P. A., Mannina, G., Cosenza, A., & Neumann, M. B., (2015). Global sensitivity analysis for urban water quality modelling: Terminology, convergence and comparison of different methods, Journal of Hydrology, 522, 339-352.

Vanuytrecht, E., Raes, D., & Willems, P., (2014). Global sensitivity analysis of yield output from the water productivity model, Environmental Modelling & Software, 51, 323-332.

Yang, I., (2007). Risk modeling of dependence among project task durations, Computer-Aided Civil and Infrastructure Engineering, 22, 419-429.